

## CONSTRUCTING FORMULAE

identify processes in calculations.

generalise using letters and simplify (where possible)

1. (a) find the cost of a car repair requiring £180 of parts and 3 hours of labour at £35 per hour.  
(b) write a formula for the cost, £C, where £p is the cost of parts and t hours of labour are charged at £n per hour.

2.

CAR HIRE CHARGES		
minimum charge:	£20 per day	
mileage charge:	first 200 miles	no charge
	over 200 miles	each extra mile 20p

- (a) Find the cost of a 6 day hire, travelling 500 miles.
- (b) Write a formula for the cost, £C, of hiring a car for d days, travelling n miles where  $n > 200$ .

3. Prices for hiring a holiday cottage are shown.

SEASON	initial cost for 4 guests	Cost for each additional guest
Summer	£300	£45
Spring/Autumn	£225	£35

(a) Find the cost of hire in Spring for 6 guests.

(b) Write a formula for the cost, £ $C$ , of  $n$  guests in Summer, where  $n$  is greater than 4.

FULLY simplify this expression.

4. The cost of posting a parcel depends on weight and delivery time.

Delivery Time	Cost
by 10am next day	£18 for 10kg + 80p for each extra kg
by noon next day	£14 for 10kg + 70p for each extra kg
by 5pm next day	£12 for 10kg + 50p for each extra kg

(a) Calculate the cost of a 16 kg parcel for delivery by noon the next day.

(b) Write a formula for the cost, £ $C$ , of a parcel weight  $w$  kg, where  $w$  is greater than 10.

The parcel is to be delivered by 10am the next day.

FULLY simplify this expression.

## PATTERNS

identify links between term and pattern.

generalise for  $n^{\text{th}}$  term and simplify (where possible)

$$5. \quad 1^3 + 2^3 = \frac{2^2 \times 3^2}{4}$$

$$1^3 + 2^3 + 3^3 = \frac{3^2 \times 4^2}{4}$$

$$1^3 + 2^3 + 3^3 + 4^3 = \frac{4^2 \times 5^2}{4}$$

Write an expression for:

(a)  $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3$

(b)  $1^3 + 2^3 + \dots + n^3$

6. The sum of consecutive odd numbers starting from 3 can be written:

$$4 \text{ terms} \quad 3+5+7+9 = 4 \times 6$$

$$5 \text{ terms} \quad 3+5+7+9+11 = 5 \times 7$$

$$6 \text{ terms} \quad 3+5+7+9+11+13 = 6 \times 8$$

(a) Write  $3+5+7+\dots+25$  in the same way.

(b) Write a formula for the sum of the first  $n$  terms of this sequence.

7. The odd numbers 1 , 3 , 5 , 7..... can be written:

$$1^{\text{st}} \text{ odd number } 1 = 1^2 - 0^2$$

$$2^{\text{nd}} \text{ odd number } 3 = 2^2 - 1^2$$

$$3^{\text{rd}} \text{ odd number } 5 = 3^2 - 2^2$$

(a) Write the 4<sup>th</sup> odd number in the same way.

(b) Express 19 in the same way.

(c) Write a formula for the n<sup>th</sup> odd number.

FULLY simplify this expression.

8. The sequence 15 , 21 , 27..... can be written:

$$1^{\text{st}} \text{ term } 15 = 4^2 - 1^2$$

$$2^{\text{nd}} \text{ term } 21 = 5^2 - 2^2$$

$$3^{\text{rd}} \text{ term } 27 = 6^2 - 3^2$$

(a) Write the 7<sup>th</sup> term in the same way.

(b) Write a formula for the n<sup>th</sup> term.

FULLY simplify the expression.

9. The sequence 4 , 8 , 12..... can be written:

$$1^{\text{st}} \text{ term} \quad 4 = 2^2 - 0^2$$

$$2^{\text{nd}} \text{ term} \quad 8 = 3^2 - 1^2$$

$$3^{\text{rd}} \text{ term} \quad 12 = 4^2 - 2^2$$

(a) Write the 9<sup>th</sup> term in the same way.

(b) Write a formula for the n<sup>th</sup> term.

FULLY simplify the expression.

10. The sequence 6 , 18 , 36..... can be written:

$$1^{\text{st}} \text{ term} \quad 6 = 2^3 - 1^3 - 1$$

$$2^{\text{nd}} \text{ term} \quad 18 = 3^3 - 2^3 - 1$$

$$3^{\text{rd}} \text{ term} \quad 36 = 4^3 - 3^3 - 1$$

(a) Write the 4<sup>th</sup> term in the same way.

(b) Write a formula for the n<sup>th</sup> term.

FULLY simplify the expression.

## CONSTRUCTING FORMULAE

identify processes in calculations.

generalise using letters and simplify (where possible)

1. (a) find the cost of a car repair requiring £180 of parts and 3 hours of labour at £35 per hour.

$$£180 + £35 \times 3 = £285$$

- (b) write a formula for the cost, £C,  
where £p is the cost of parts and  
t hours of labour are charged at £n per hour.

$$C = p + nt$$

2.

### CAR HIRE CHARGES

minimum charge: £20 per day

mileage charge: first 200 miles no charge  
over 200 miles each extra mile 20p

- (a) Find the cost of a 6 day hire, travelling 500 miles.

$$£20 \times 6 + £0.2 \times (500 - 200) = £180$$

- (b) Write a formula for the cost, £C, of hiring a car for d days, travelling n miles where  $n > 200$ .

$$C = 20d + 0.2(n - 200)$$

3. Prices for hiring a holiday cottage are shown.

SEASON	initial cost for 4 guests	Cost for each additional guest
Summer	£300	£45
Spring/Autumn	£225	£35

(a) Find the cost of hire in Spring for 6 guests.

$$£225 + £35 \times (6 - 4) = £295$$

(b) Write a formula for the cost, £C, of n guests in Summer, where n is greater than 4.

$$C = 300 + 45 \times (n - 4)$$

FULLY simplify this expression.

$$= 300 + 45n - 180$$

$$C = 45n + 120$$

4. The cost of posting a parcel depends on weight and delivery time.

Delivery Time	Cost
by 10am next day	£18 for 10kg + 80p for each extra kg
by noon next day	£14 for 10kg + 70p for each extra kg
by 5pm next day	£12 for 10kg + 50p for each extra kg

(a) Calculate the cost of a 16 kg parcel for delivery by noon the next day.

$$£14 + £0.70 \times (16 - 10) = £18.20$$

(b) Write a formula for the cost, £C, of a parcel weight w kg, where w is greater than 10.

$$C = 18 + 0.80 \times (w - 10)$$

The parcel is to be delivered by 10am the next day.

$$= 18 + 0.8w - 8$$

FULLY simplify this expression.

$$C = 0.8w + 10$$



## PATTERNS

identify links between term and pattern.

generalise for  $n^{\text{th}}$  term and simplify (where possible)

$$5. \quad 1^3 + 2^3 = \frac{2^2 \times 3^2}{4}$$

$$1^3 + 2^3 + 3^3 = \frac{3^2 \times 4^2}{4}$$

$$1^3 + 2^3 + 3^3 + 4^3 = \frac{4^2 \times 5^2}{4}$$

Write an expression for:

$$(a) \quad 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = \frac{7^2 \times 8^2}{4}$$

$$(b) \quad 1^3 + 2^3 + \dots + n^3 = \frac{n^2 (n+1)^2}{4}$$

6. The sum of consecutive odd numbers starting from 3 can be written:

$$4 \text{ terms} \quad 3+5+7+9 = 4 \times 6$$

$$5 \text{ terms} \quad 3+5+7+9+11 = 5 \times 7$$

$$6 \text{ terms} \quad 3+5+7+9+11+13 = 6 \times 8$$

(a) Write  $3+5+7+\dots+25$  in the same way.  $12 \times 14$

(b) Write a formula for the sum of the first  $n$  terms of this sequence.  $n(n+2)$



7. The odd numbers 1, 3, 5, 7..... can be written:

$$1^{\text{st}} \text{ odd number } 1 = 1^2 - 0^2$$

$$2^{\text{nd}} \text{ odd number } 3 = 2^2 - 1^2$$

$$3^{\text{rd}} \text{ odd number } 5 = 3^2 - 2^2$$

$$7 = 4^2 - 3^2$$

(a) Write the 4<sup>th</sup> odd number in the same way.

(b) Express 19 in the same way.  $19 = 10^2 - 9^2$

(c) Write a formula for the n<sup>th</sup> odd number.

FULLY simplify this expression.

$$\begin{aligned} & n^2 - (n - 1)^2 \\ &= n^2 - (n^2 - 2n + 1) \\ &= n^2 - n^2 + 2n - 1 \\ &= 2n - 1 \end{aligned}$$

8. The sequence 15, 21, 27..... can be written:

$$1^{\text{st}} \text{ term } 15 = 4^2 - 1^2$$

$$2^{\text{nd}} \text{ term } 21 = 5^2 - 2^2$$

$$3^{\text{rd}} \text{ term } 27 = 6^2 - 3^2$$

(a) Write the 7<sup>th</sup> term in the same way.  $51 = 10^2 - 7^2$

(b) Write a formula for the n<sup>th</sup> term.

FULLY simplify the expression.

$$\begin{aligned} & (n+3)^2 - n^2 \\ &= n^2 + 6n + 9 - n^2 \\ &= 6n + 9 \end{aligned}$$

9. The sequence 4 , 8 , 12..... can be written:

$$1^{\text{st}} \text{ term} \quad 4 = 2^2 - 0^2$$

$$2^{\text{nd}} \text{ term} \quad 8 = 3^2 - 1^2$$

$$3^{\text{rd}} \text{ term} \quad 12 = 4^2 - 2^2$$

(a) Write the 9<sup>th</sup> term in the same way.  $36 = 10^2 - 8^2$

(b) Write a formula for the n<sup>th</sup> term.

FULLY simplify the expression.

$$\begin{aligned} & (n+1)^2 - (n-1)^2 \\ &= n^2+2n+1 - (n^2-2n+1) \\ &= n^2+2n+1 - n^2+2n-1 \\ &= 4n \end{aligned}$$

10. The sequence 6 , 18 , 36..... can be written:

$$1^{\text{st}} \text{ term} \quad 6 = 2^3 - 1^3 - 1$$

$$2^{\text{nd}} \text{ term} \quad 18 = 3^3 - 2^3 - 1$$

$$3^{\text{rd}} \text{ term} \quad 36 = 4^3 - 3^3 - 1$$

(a) Write the 4<sup>th</sup> term in the same way.

$$60 = 5^3 - 4^3 - 1$$

(b) Write a formula for the n<sup>th</sup> term.

FULLY simplify the expression.

$$\begin{aligned} & (n+1)^3 - n^3 - 1 \\ &= n^3+3n^2+3n+1 - n^3 - 1 \\ &= 3n^2 + 3n \\ &= 3n(n+1) \end{aligned}$$

$$\begin{aligned} & (n+1)^3 \\ &= (n+1)(n+1)(n+1) \\ &= (n+1)(n^2+2n+1) \\ &= n(n^2+2n+1) + 1(n^2+2n+1) \\ &= n^3+2n^2+n + n^2+2n+1 \\ &= n^3+3n^2+3n+1 \end{aligned}$$