HIGHER MATHEMATICS
COURSE NOTES

UNIT 3

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

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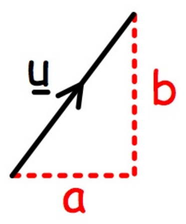
VECTORS

SCALAR quantities have SIZE (magnitude).

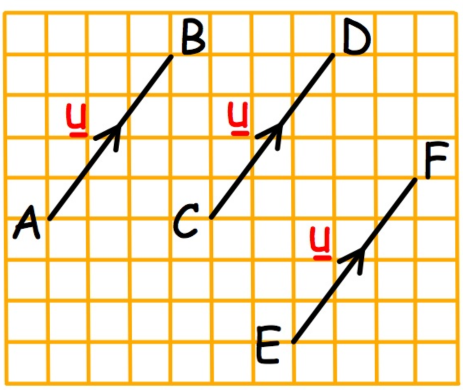
VECTOR quantities have SIZE and DIRECTION.

DIRECTED LINE SEGMENT

A line of a particular size and direction is used to represent a vector.

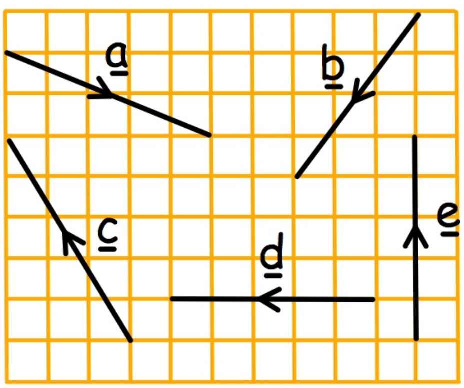


COMPONENT FORM $\underline{u} = \begin{pmatrix} a \\ b \end{pmatrix}$



$\overrightarrow{AB} = \overrightarrow{CD} = \overrightarrow{EF} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

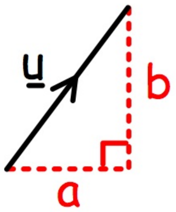
Three directed line segments,
same size and direction,
same component form,
same vector **u**. $\underline{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$



$\underline{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

$\underline{c} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$ $\underline{d} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$ $\underline{e} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

MAGNITUDE   Follows from Pyth. Thm.

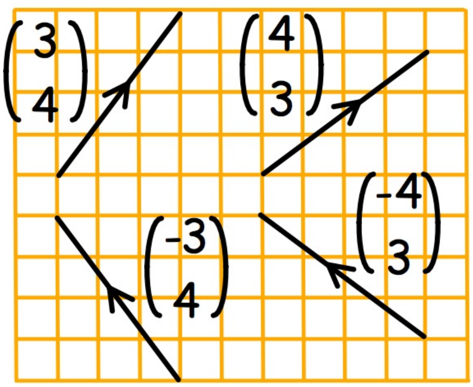


$\underline{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ $|\underline{u}| = \sqrt{a^2 + b^2}$

$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

$$|\overrightarrow{AB}| = \sqrt{(-3)^2 + 6^2}$$
$$= \sqrt{45}$$
$$= \underline{\underline{3\sqrt{5} \text{ units}}}$$

NOTE: different vectors can have the same magnitude.

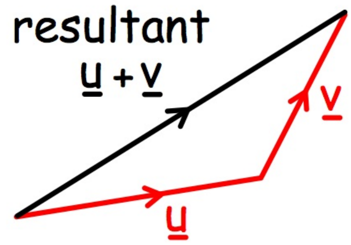


all different vectors
same magnitude 5 units.

ADD and SUBTRACT

By "head-to-tail" triangle.

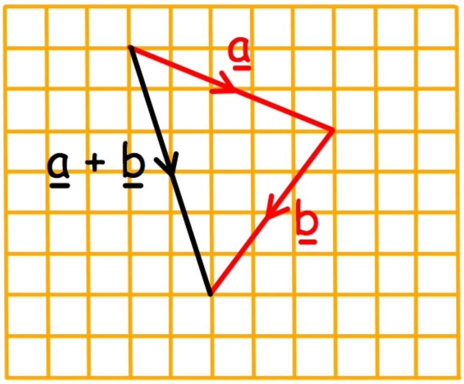
NOTE: $|\underline{u}| + |\underline{v}| > |\underline{u}+\underline{v}|$



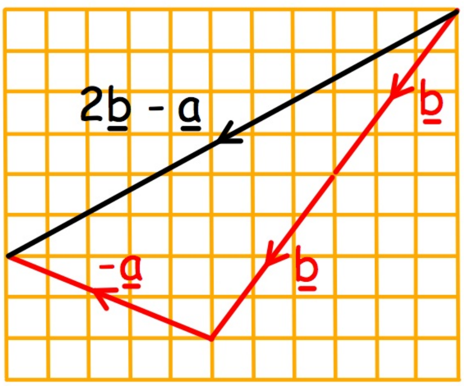
By components: add or subtract components.

MULTIPLY BY A SCALAR: multiply components.

$$k\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}ka\\kb\end{pmatrix}$$



$$\underline{a} + \underline{b} = \begin{pmatrix}5\\-2\end{pmatrix} + \begin{pmatrix}-3\\-4\end{pmatrix} = \begin{pmatrix}2\\-6\end{pmatrix}$$

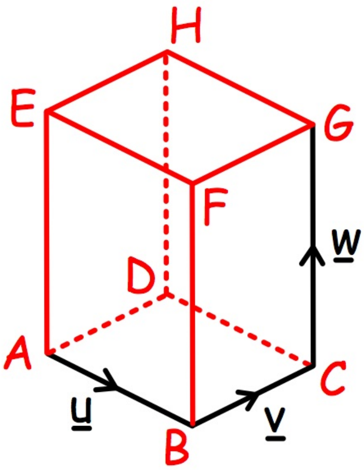


$$2\underline{b} - \underline{a} = 2\begin{pmatrix}-3\\-4\end{pmatrix} - \begin{pmatrix}5\\-2\end{pmatrix}$$

$$= \begin{pmatrix}-6\\-8\end{pmatrix} - \begin{pmatrix}5\\-2\end{pmatrix}$$

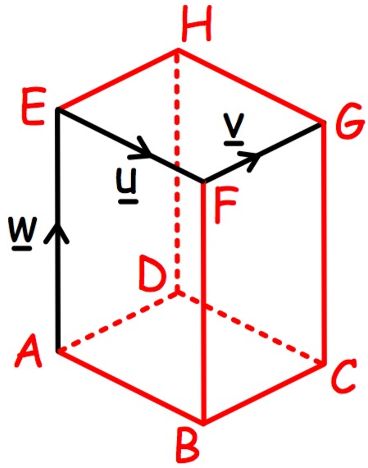
$$= \begin{pmatrix}-11\\-6\end{pmatrix}$$

3D VECTORS



$\overrightarrow{AG} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CG}$

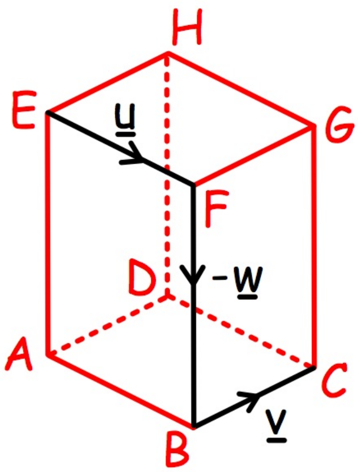
$= \underline{u} + \underline{v} + \underline{w}$



same result regardless of 'route'

$\overrightarrow{AG} = \overrightarrow{AE} + \overrightarrow{EF} + \overrightarrow{FG}$

$= \underline{w} + \underline{u} + \underline{v}$



subtraction is adding the negative

$\overrightarrow{EC} = \overrightarrow{EF} + \overrightarrow{FB} + \overrightarrow{BC}$

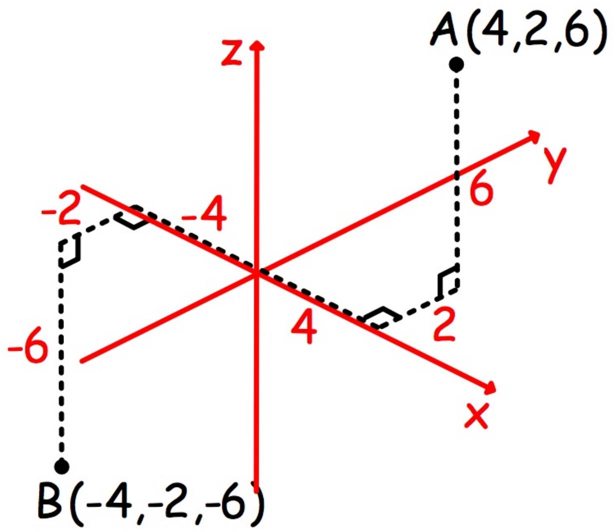
$= \overrightarrow{EF} - \overrightarrow{BF} + \overrightarrow{BC}$

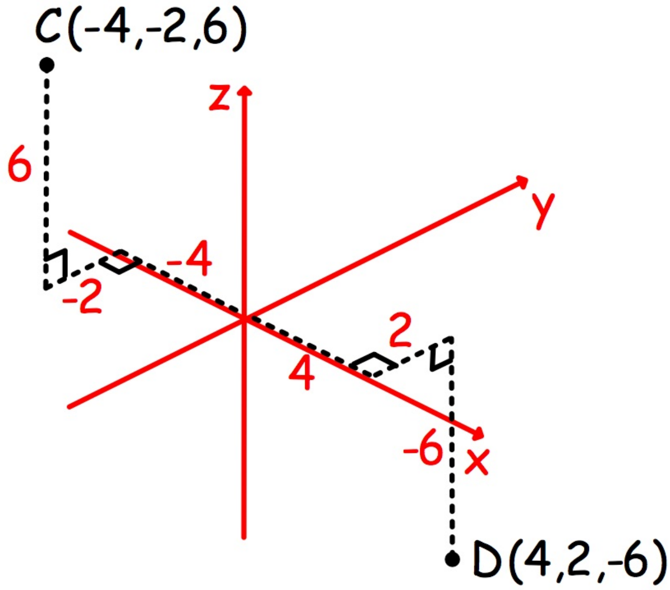
$= \underline{u} - \underline{w} + \underline{v}$

NEGATIVE: direction reversed $\overrightarrow{FB} = -\overrightarrow{BF}$

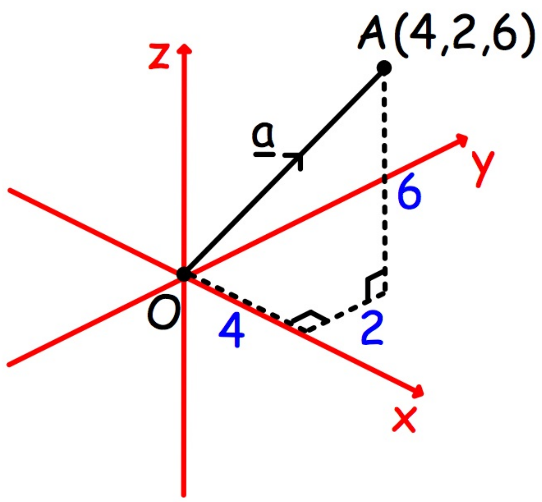
3D COORDINATES

Points (x, y, z) plotted on 3 mutually perpendicular axes.



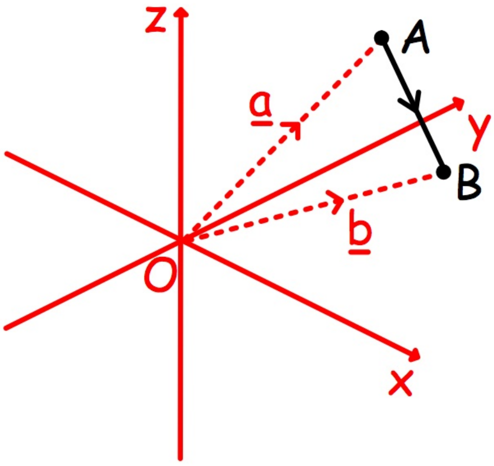


The POSITION VECTOR of point A is given by $\overrightarrow{OA}$.



$$\underline{a} = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}$$

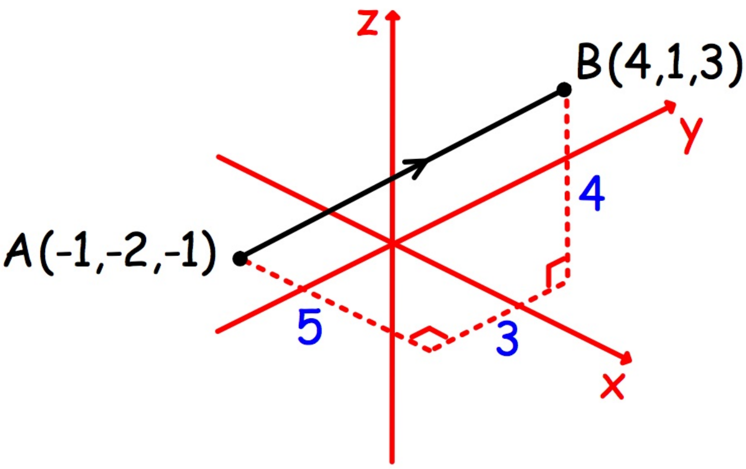
POSITION VECTORS



$$\overrightarrow{AB} = \underline{b} - \underline{a}$$

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$



$$\overrightarrow{AB} = \underline{b} - \underline{a}$$

$$= \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}$$

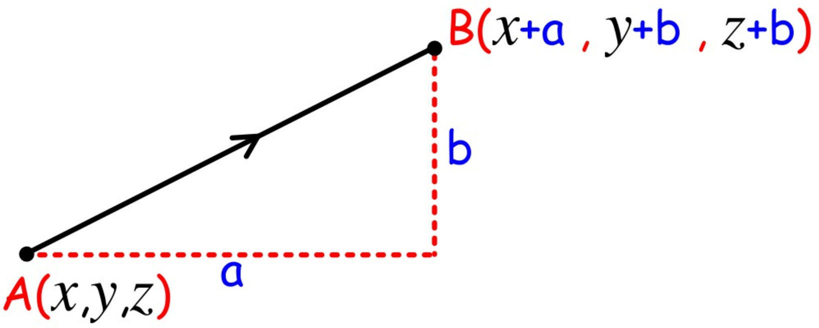
$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix}$$

OR

$$A(-1, -2, -1) \xrightarrow{\;+5,\ +3,\ +4\;} B(4, 1, 3)$$

TRANSLATION $\overrightarrow{AB}$ represents a movement from A to B.

$$\overrightarrow{AB} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

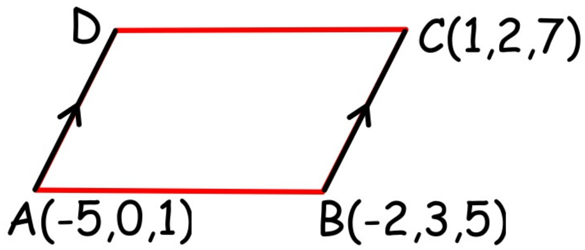


(1) If $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix}$ and A(-1,-2,-1), find the coordinates of B.

A(-1,-2,-1) ⟶ B(-1 + 5 , -2 + 3 , -1 + 4)

<u>B(4,1,3)</u>

(2) For parallelogram ABCD,
find the coordinates of D.



+3 -1 +2

B(-2 , 3 , 5) ⟶ C(1 , 2 , 7)

$$\overrightarrow{BC} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

parallelogram: $\overrightarrow{AD} = \overrightarrow{BC} \Rightarrow \overrightarrow{AD} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

+3 -1 +2

A(-5 , 0 , 1) ⟶ D

<u>D(-2,-1,3)</u>

MAGNITUDE   Follows from Pyth. Thm.

$$\underline{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad |\underline{u}| = \sqrt{a^2 + b^2 + c^2}$$

Find the distance from A(-2,3,5) to B(1,2,7).

$$\vec{AB} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \qquad |\vec{AB}| = \sqrt{3^2 + (-1)^2 + 2^2}$$

$$= \sqrt{14} \text{ units}$$

NOTE: can use this instead of the Distance Formula.

UNIT VECTOR: has a magnitude of 1.

If $\underline{u} = \begin{pmatrix} 1/2 \\ a \\ -1/2 \end{pmatrix}$ is a unit vector, find the value of a.

$|\underline{u}| = 1$

$$|\underline{u}|^2 = (1/2)^2 + a^2 + (-1/2)^2$$

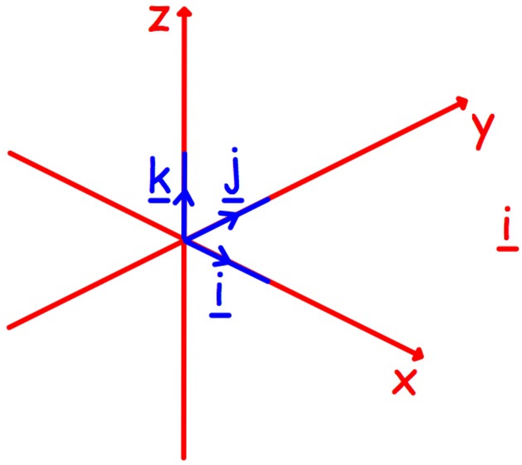
$$1 = 1/4 + a^2 + 1/4$$

$$a^2 = 1/2$$

$$a = \pm 1/\sqrt{2}$$

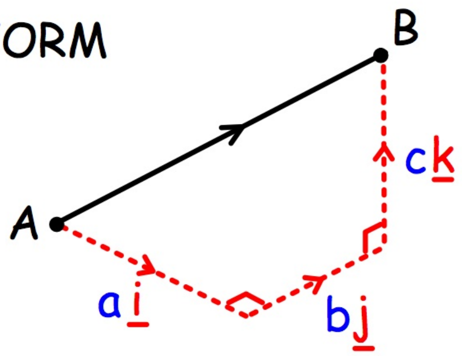
BASIS VECTORS

Three unit vectors $\underline{i}$, $\underline{j}$ and $\underline{k}$ in the OX, OY and OZ directions are used as the basis of 3 dimensional space.



$$\underline{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \underline{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \underline{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$\underline{i}$, $\underline{j}$, $\underline{k}$ FORM



$$\vec{AB} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\vec{AB} = a\underline{i} + b\underline{j} + c\underline{k}$$

$$\begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} = 2\underline{i} - 3\underline{j} + 5\underline{k} \qquad \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} = \underline{i} - 3\underline{j} - 2\underline{k}$$

$$\begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix} = 4\underline{i} - \underline{k} \qquad \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} = 5\underline{i} + 3\underline{j}$$

ADD and SUBTRACT: add or subtract components.

MULTIPLY BY A SCALAR: multiply components.

$$k\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} ka \\ kb \\ kc \end{pmatrix}$$

If $\underline{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$, find $|\underline{b} - 2\underline{a}|$.

$$\underline{b} - 2\underline{a} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -4 \end{pmatrix}$$

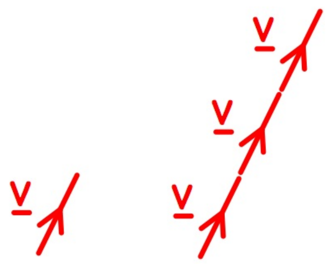
$$|\underline{b} - 2\underline{a}| = \sqrt{(-3)^2 + 4^2 + (-4)^2} = \underline{\underline{\sqrt{41} \text{ units}}}$$

PARALLEL:

$$k\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} ka \\ kb \\ kc \end{pmatrix}$$

$\underline{u} = k\underline{v}$

$\Rightarrow$ $\underline{u}$ and $\underline{v}$ are parallel



$\overrightarrow{AB} = \begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$. Show AB is parallel to CD.

OR

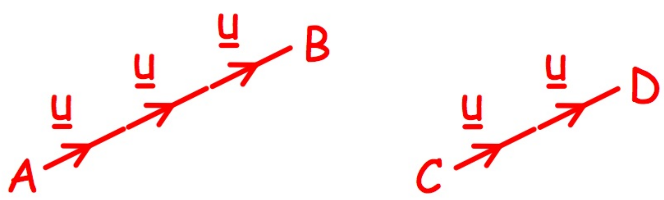
$\overrightarrow{AB} = {}^3/_2\,\overrightarrow{CD}$

$\overrightarrow{CD} = {}^2/_3\,\overrightarrow{AB}$

$\overrightarrow{AB} = 3\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ $\overrightarrow{CD} = 2\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

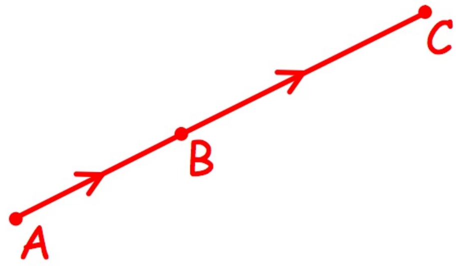
$2\overrightarrow{AB} = 3\overrightarrow{CD}$

$\Rightarrow$ AB and CD are parallel



COLLINEAR POINTS: points lie on the same line.

$\overrightarrow{AB} = k\,\overrightarrow{BC}$



$\Rightarrow$ lines AB and BC are parallel and share common point B

$\Rightarrow$ points A, B and C are collinear

NOTE: lines parallel, points collinear

Show points A(-8,3,-7) , B(1,0,-1) and C(7,-2,3) are collinear and find the ratio AB:BC.

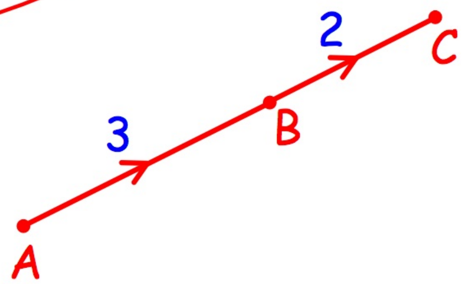
$$\overrightarrow{AB} = \begin{pmatrix} 9 \\ -3 \\ 6 \end{pmatrix} = 3\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \qquad \overrightarrow{BC} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

OR

$\overrightarrow{AB} = {}^{3}/_{2}\,\overrightarrow{BC}$

$\overrightarrow{BC} = {}^{2}/_{3}\,\overrightarrow{AB}$

$2\overrightarrow{AB} = 3\,\overrightarrow{BC}$



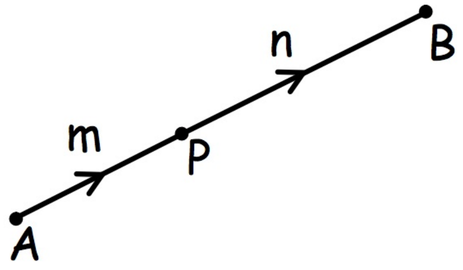
$\Rightarrow$ lines AB and BC are parallel and share common point B

$\Rightarrow$ points A, B and C are collinear

AB:BC = 3:2

DIVIDING A LINE

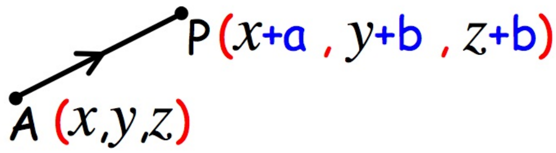
$\overrightarrow{AP}$ is a fraction of $\overrightarrow{AB}$:



(i) find $\overrightarrow{AB}$

(ii) find $\overrightarrow{AP}$ $\qquad \overrightarrow{AP} = \frac{m}{m+n}\,\overrightarrow{AB}$

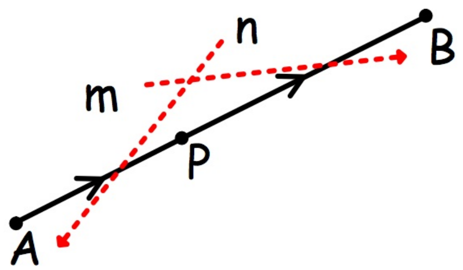
(iii) find P $\qquad \overrightarrow{AP} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$



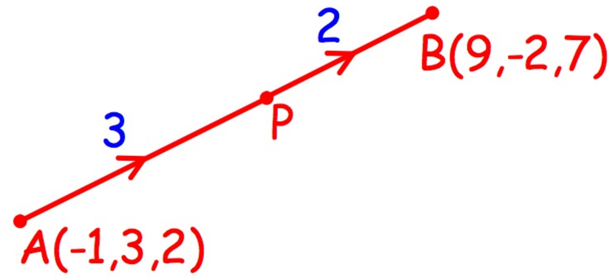
SECTION FORMULA

uses position vectors:

$$\underline{p} = \frac{m\underline{b} + n\underline{a}}{m + n}$$



(1) A(-1,3,2), B(9,-2,7). Find the coordinates of point P, which divides AB internally in the ratio 3:2 .



$$\overrightarrow{AB} = \begin{pmatrix} 10 \\ -5 \\ 5 \end{pmatrix}$$

$$\overrightarrow{AP} = {}^{3}/_{5}\ \overrightarrow{AB} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$$

+6 -3 +3

A(-1 , 3 , 2) ⟶ P(5 , 0 , 5)

OR

$$\underline{p} = \frac{3\underline{b} + 2\underline{a}}{3 + 2}$$

$$= \frac{1}{5}\left[3\begin{pmatrix} 9 \\ -2 \\ 7 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} \right]$$

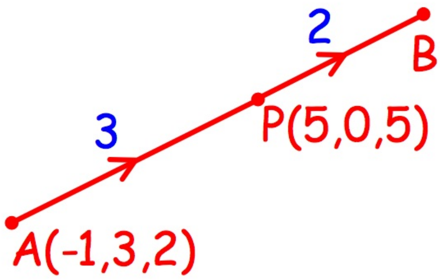
$$= \frac{1}{5}\begin{pmatrix} 25 \\ 0 \\ 25 \end{pmatrix}$$

$$\underline{p} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$$

P(5 , 0 , 5)

(2) A(-1,3,2), P(5,0,5). Line AP is produced $^2/_3$ of its length to point B. Find the coordinates of B.

$$\vec{AP} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$$



$$\vec{PB} = {}^2/_3 \ \vec{AP} \quad = \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}$$

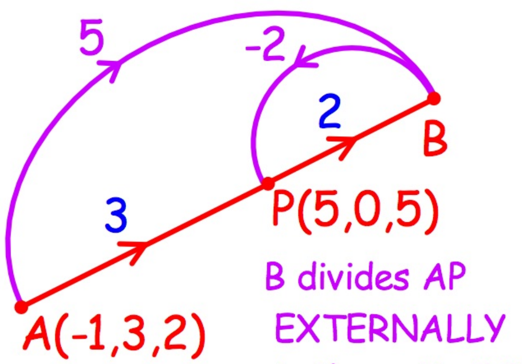
+4 -2 +2

P(5 , 0 , 5) ⟶ B(9 , -2 , 7)

OR

$$\underline{b} = \frac{5\underline{p} - 2\underline{a}}{5 + (-2)}$$

$$= \frac{1}{3}\left[5\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} \right]$$



$$= \frac{1}{3}\begin{pmatrix} 27 \\ -6 \\ 21 \end{pmatrix}$$

$$\underline{b} = \begin{pmatrix} 9 \\ -2 \\ 7 \end{pmatrix}$$

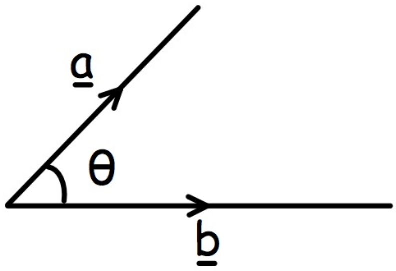
B(9 , -2 , 7)

SCALAR PRODUCT (DOT PRODUCT)

Multiply two vectors for a scalar result.

$\underline{a}.\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$ $\quad \underline{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

$\underline{a}.\underline{b} = |\underline{a}||\underline{b}|\cos\theta \, , \; \underline{a} \neq \underline{0} \, , \; \underline{b} \neq \underline{0}$



NOTE: (i) vectors "pull away" from each other.
(ii) $\underline{a}.\underline{b} = \underline{b}.\underline{a}$

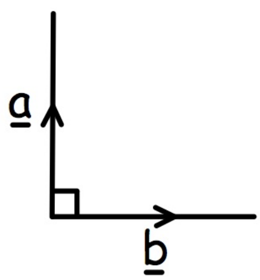
$\underline{a}.\underline{b}$ can be positive, zero or negative depending on θ.

θ acute $\Rightarrow$ $\underline{a}.\underline{b}$ positive

θ obtuse $\Rightarrow$ $\underline{a}.\underline{b}$ negative

$\theta = 90°$ $\quad \underline{a}.\underline{b} = 0$

$\Rightarrow$ $\underline{a}$ is perpendicular to $\underline{b}$



(1) $\underline{p} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ $\underline{q} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$

$$\underline{a}.\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$$

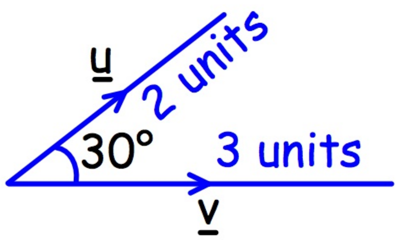
$$\underline{p}.\underline{q} = -2\times4 + 3\times0 + 1\times3$$
$$= -8 + 0 + 3$$
$$\underline{p}.\underline{q} = -5$$

(2) $\underline{r} = \begin{pmatrix} -5 \\ -1 \\ 2 \end{pmatrix}$ $\underline{s} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$

$$\underline{r}.\underline{s} = -5\times2 + (-1)\times(-2) + 2\times4$$
$$= -10 + 2 + 8$$
$$\underline{r}.\underline{s} = 0$$

NOTE: $\Rightarrow$ $\underline{r}$ is perpendicular to $\underline{s}$

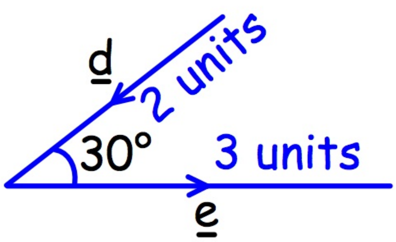
(3)



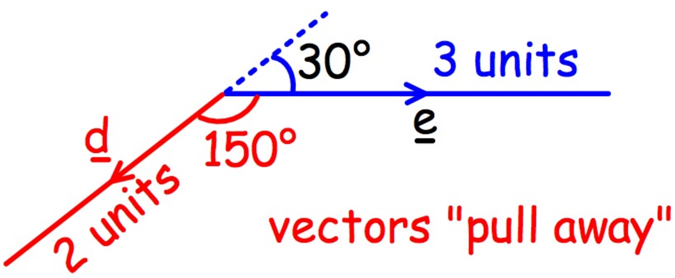
$$\underline{a}.\underline{b} = |\underline{a}||\underline{b}|\cos\theta$$

$$\underline{u}.\underline{v} = 2 \times 3 \times \cos 30°$$
$$= 2 \times 3 \times \sqrt{3}/2$$
$$\underline{u}.\underline{v} = 3\sqrt{3}$$

(4)



$$\underline{d}.\underline{e} = 2 \times 3 \times \cos 150°$$
$$= 2 \times 3 \times (-\sqrt{3}/2)$$
$$\underline{d}.\underline{e} = -3\sqrt{3}$$



(5) $\underline{u} = -3\underline{i} + 3\underline{j} + 3\underline{k}$ $\quad \underline{v} = \underline{i} + 5\underline{j} - \underline{k}$

Show that $\underline{u}+\underline{v}$ is perpendicular to $\underline{u}-\underline{v}$.

$$\underline{u} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} \qquad \underline{v} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$$

$$\underline{u}+\underline{v} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix} \qquad \underline{u}-\underline{v} = \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix}$$

$$\begin{aligned}(\underline{u}+\underline{v}).(\underline{u}-\underline{v}) &= -2\times(-4) + 8\times(-2) + 2\times4 \\ &= 8 + (-16) + 8 \\ &= 0\end{aligned}$$

$(\underline{u}+\underline{v}).(\underline{u}-\underline{v}) = 0$

$\Rightarrow$ $\underline{u}+\underline{v}$ is perpendicular to $\underline{u}-\underline{v}$

(6) $\underline{m} = \begin{pmatrix} -1 \\ k \\ -2 \end{pmatrix}$ $\quad \underline{n} = \begin{pmatrix} -4 \\ 2 \\ 5 \end{pmatrix}$

Find k if $\underline{m}$ and $\underline{n}$ are perpendicular.

$$\begin{aligned}\underline{m}.\underline{n} &= -1\times(-4) + k\times2 + -2\times5 \\ &= 4 + 2k + (-10) \\ &= 2k - 6\end{aligned}$$

$\underline{m}$ is perpendicular to $\underline{n}$

$\Rightarrow$ $\underline{m}.\underline{n} = 0$

$2k - 6 = 0$

$k = 3$

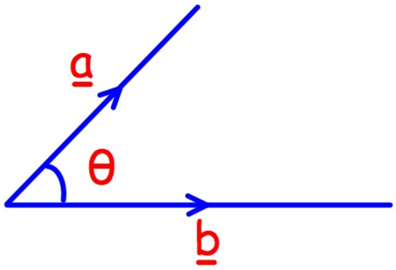
ANGLE BETWEEN VECTORS

Combining formulae:

$$\underline{a}.\underline{b} = |\underline{a}||\underline{b}|\cos\theta$$

$$\underline{a}.\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{|\underline{a}||\underline{b}|}$$



NOTE: vectors "pull away"

(1) $\underline{a} = 2\underline{i} + 2\underline{j} + \underline{k}$ $\quad\quad$ $\underline{b} = 2\underline{i} + 2\underline{k}$

Find the angle between the vectors.

$$\underline{a} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad\quad \underline{b} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

$$|\underline{a}| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9}$$

$$|\underline{b}| = \sqrt{2^2 + 0^2 + 2^2} = \sqrt{8}$$

$$\underline{a}.\underline{b} = 2\times2 + 2\times0 + 1\times2 = 6$$

$$\cos\theta = \frac{6}{(\sqrt{9}\times\sqrt{8})}$$

$$\frac{6}{\sqrt{72}} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\underline{\underline{\theta = 45°}}$$

(2) A(-4,2,5), B(-3,0,4), C(-2,0,1). Find $\angle ABC$.

vectors "pull away" from angle at B

$$\overrightarrow{BA} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \qquad |\overrightarrow{BA}| = \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6}$$

$$\overrightarrow{BC} = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} \qquad |\overrightarrow{BC}| = \sqrt{1^2 + 0^2 + (-3)^2} = \sqrt{10}$$

$\underline{a}.\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$

$$\overrightarrow{BA}.\overrightarrow{BC} = -1 \times 1 + 2 \times 0 + 1 \times (-3)$$

$$= -1 + 0 + (-3)$$

$$\overrightarrow{BA}.\overrightarrow{BC} = -4$$

$\underline{a}.\underline{b} = |\underline{a}||\underline{b}|\cos\theta$

$$\overrightarrow{BA}.\overrightarrow{BC} = |\overrightarrow{BA}||\overrightarrow{BC}|\cos B$$

$$-4 = \sqrt{6} \times \sqrt{10} \times \cos B$$

$\cos\theta = \dfrac{\underline{a}.\underline{b}}{|\underline{a}||\underline{b}|}$

$$\cos B = \frac{-4}{(\sqrt{6} \times \sqrt{10})}$$

$$\cos B = -0.51369...$$

$$B = 121.0909...$$

$$\underline{\underline{\angle ABC \approx 121.1^\circ}}$$

PROPERTIES OF THE SCALAR PRODUCT

$\underline{a}.\underline{a} = |\underline{a}|^2$ since $|\underline{a}||\underline{a}|\cos 0° = |\underline{a}||\underline{a}| \times 1$

$\underline{a}.\underline{b} = 0 \Rightarrow \underline{a}$ is perpendicular to $\underline{b}$

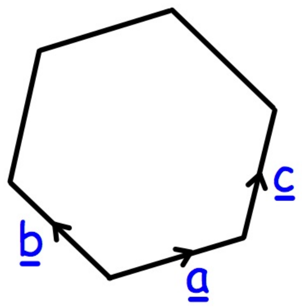
$\underline{a}.\underline{b} = \underline{b}.\underline{a}$

$\underline{a}.(\underline{b} + \underline{c}) = \underline{a}.\underline{b} + \underline{a}.\underline{c}$

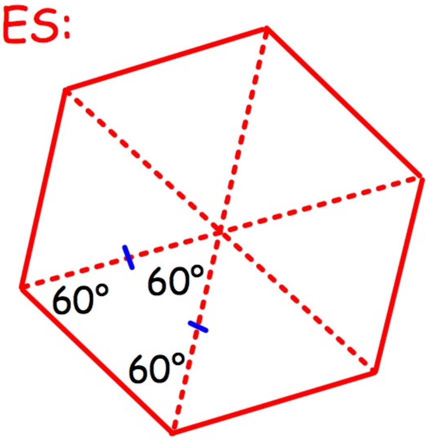
Regular hexagon side 2 units.

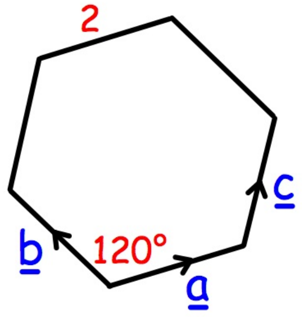
(a) Find $\underline{a}.(\underline{b} + \underline{c})$ and comment on the result.

(b) Find $\underline{b}.(\underline{a} + \underline{b} + \underline{c})$.

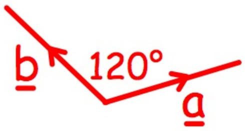


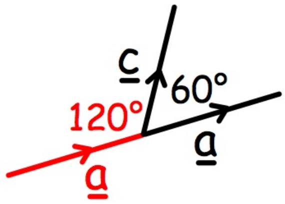
ANGLES:





(a)





$\underline{a}.\underline{b} = |\underline{a}||\underline{b}|\cos 120°$

$= 2 \times 2 \times (-1/2)$

$\underline{a}.\underline{b} = -2$

$\underline{a}.\underline{c} = |\underline{a}||\underline{c}|\cos 60°$

$= 2 \times 2 \times 1/2$

$\underline{a}.\underline{c} = 2$

$\underline{a}.(\underline{b} + \underline{c})$

$= \underline{a}.\underline{b} + \underline{a}.\underline{c}$

$= -2 + 2$

$= 0$

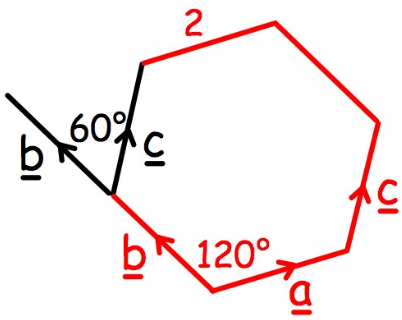
$\underline{a}.(\underline{b} + \underline{c}) = 0$

$\Rightarrow$ $\underline{a}$ is perpendicular to $\underline{b} + \underline{c}$

(b) $\underline{b}.\underline{c} = |\underline{b}||\underline{c}|\cos 60°$

$= 2 \times 2 \times 1/2$

$\underline{b}.\underline{c} = 2$



$\underline{b}.\underline{a} = \underline{a}.\underline{b} = -2$

$\underline{b}.\underline{b} = |\underline{b}|^2 = 2^2 = 4$ since $|\underline{b}||\underline{b}|\cos 0° = |\underline{b}||\underline{b}| \times 1$

$\underline{b}.(\underline{a} + \underline{b} + \underline{c})$

$= \underline{b}.\underline{a} + \underline{b}.\underline{b} + \underline{b}.\underline{c}$

$= -2 + 4 + 2$

$= \underline{\underline{4}}$