

MATHEMATICS
S1+S2 COURSE NOTES
LEVEL 3/4

I



MATHS

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CHAPTER 1: NUMBER

POWERS

$$\begin{array}{c} \text{index} \nearrow \\ a^n \\ \text{base} \nearrow \end{array} = a \times a \times \dots \times a \text{ to } n \text{ terms}$$

" 2 to the power of 5 "

(i) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

(ii) $2^7 = 2^5 \times 2 \times 2 = 32 \times 2 \times 2 = 128$

(1) 2^3
 $= 2 \times 2 \times 2$
 $= 8$

(2) $(7 - 3)^2$
 $= 4^2$
 $= 4 \times 4$
 $= 16$

(3) $7^2 - 3^2$
 $= 7 \times 7 - 3 \times 3$
 $= 49 - 9$
 $= 40$

SQUARE ROOT

Square and square root are inverse processes.

$$\begin{array}{l} 3^2 = 9 \\ \sqrt{9} = 3 \end{array}$$

SUM (add)

$$12 + 4 = 16$$

DIFFERENCE (subtract)

$$12 - 4 = 8$$

PRODUCT (multiply)

$$12 \times 4 = 48$$

QUOTIENT (divide)

$$12 \div 4 = 3$$

MULTIPLE:

multiples of 6: 6, 12, 18, 24 ...

multiples of 8: 8, 16, 24, 36 ...

lowest common multiple: 24

LCM = 24

FACTOR: pairs multiply to give the number

$$12: \quad 1 \times 12 = 2 \times 6 = 3 \times 4$$

factors of 12: 1, 2, 3, 4, 6, 12

factors of 18: 1, 2, 3, 6, 9, 18

highest common factor: 6 HCF = 6

PRIME NUMBER: has exactly 2 factors, itself and 1.

2, 3, 5, 7, 11, 13, 17... **Note: 2 is the only even prime**

PRODUCT OF PRIME FACTORS:

factors of 12: 1, 2, 3, 4, 6, 12

prime factors of 12: 2, 3

product of prime factors: $12 = 2 \times 2 \times 3$

divide successively by primes to reduce the number to 1.

2	240
2	120
2	60
2	30
3	15
5	5
	1

$$\begin{aligned} 240 &= 2 \times 2 \times 2 \times 2 \times 3 \times 5 \\ &= 2^4 \times 3 \times 5 \quad \text{INDEX FORM} \end{aligned}$$

DIRECTED NUMBERS

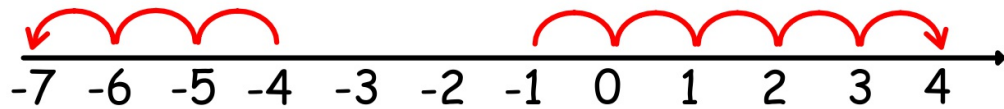
Add/Subtract

$$a + (-b) = a - b$$

$$a - (-b) = a + b$$

$$-4 + (-3) = -4 - 3 = -7$$

$$-1 - (-5) = -1 + 5 = 4$$



Multiply and Divide

Order of Calculation

$P \times \text{or } \div P = P$
$N \times \text{or } \div N = P$
$P \times \text{or } \div N = N$
$N \times \text{or } \div P = N$

\times and \div before $+$ or $-$ change the order with brackets: BRACKETS FIRST!
--

EVALUATE EXPRESSIONS

$$a = -2, b = 3, c = -5$$

$$(1) \quad b^2 + ac$$

$$= 3 \times 3 + (-2) \times (-5)$$

$$= 9 + 10$$

$$= 19$$

$$(2) \quad a(b + c)$$

$$= -2 \times (3 + (-5))$$

$$= -2 \times (-2)$$

$$= 4$$

$$(3) \quad \frac{c - b}{c + b}$$

$$= \frac{-5 - 3}{-5 + 3}$$

$$= \frac{-8}{-2}$$

$$= 4$$

$$(4) \quad 2c^2$$

$$= 2 \times (-5) \times (-5)$$

$$= 2 \times 25$$

$$= 50$$

$$(5) \quad (2c)^2$$

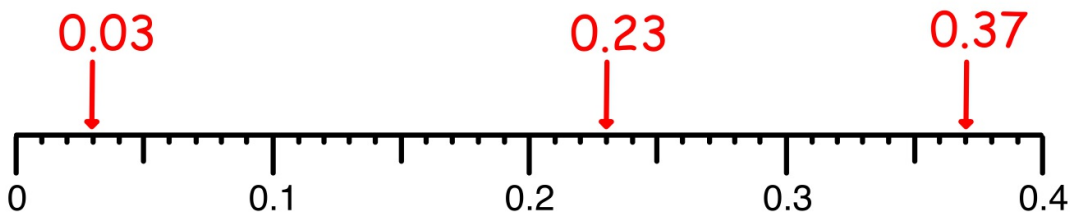
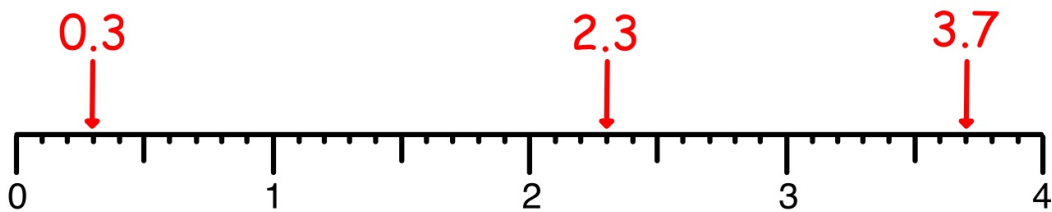
$$= (2 \times (-5))^2$$

$$= (-10)^2$$

$$= 100$$

CHAPTER 2: DECIMALS

READING SCALES



DECIMAL PLACES

the number of figures after the decimal point.

ROUNDING

(i) $12.\overset{\downarrow}{6}49$ (3DP) $\begin{cases} 12.7 \\ \text{-----} 12.65 \\ 12.6 \end{cases}$
 $= 12.6$ (1DP)

(ii) $3.\overset{\downarrow}{6}851$ (4DP) $\begin{cases} 3.69 \\ \text{-----} 3.685 \\ 3.68 \end{cases}$
 $= 3.69$ (2DP)

SIGNIFICANT FIGURES

Indicate the accuracy of a measurement.

eg. 3400 cm = 34 m = 0.034 km

Same measurement, same accuracy: each 2 significant figures

Count the number of figures used but **not** zeros:
at the **end** of a **whole** number,
at the **start** of a **decimal**.

These zeros simply keep the place value of the digits.

Examples:

0.030 2 sig fig

30100 3 sig fig

5.030 4 sig fig

30100.0 6 sig fig

ROUNDING:

Round to the place value of the number of significant figures required

Examples: rounding to 2 significant figures.

$$(i) \begin{array}{ccccccc} & \text{TH} & \text{H} & \text{T} & \text{U} & + & \\ (i) & 5 & 7 & 5 & 1 & . & 4 = 5800 \end{array} \quad \text{nearest HUNDRED}$$

↑

$$(ii) \begin{array}{ccccccc} & \text{U} & + & \text{h} & \text{th} & & \\ (ii) & 0 & . & 0 & 5 & 7 & 514 = 0.058 \end{array} \quad \text{nearest THOUSANDTH}$$

↑

NOTE: 0.058000 is wrong

multiply by 10s

$$(i) \quad 3.\overset{\curvearrowright}{4}\overset{\curvearrowright}{2} \quad \times \quad 10 \quad = \quad 34.2$$

$$(ii) \quad 5.\overset{\curvearrowright}{8}\overset{\curvearrowright}{7} \quad \times \quad 100 \quad = \quad 587$$

$$(iii) \quad 0.\overset{\curvearrowright}{0}\overset{\curvearrowright}{6}2 \quad \times \quad 100 \quad = \quad 6.2$$

divide by 10s

$$(i) \quad 3\overset{\curvearrowleft}{4}.\overset{\curvearrowleft}{2} \quad \div \quad 10 \quad = \quad 3.42$$

$$(ii) \quad 5\overset{\curvearrowleft}{8}\overset{\curvearrowleft}{7}.\quad \div \quad 100 \quad = \quad 5.87$$

$$(iii) \quad 0\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{6}.\overset{\curvearrowleft}{2} \quad \div \quad 100 \quad = \quad 0.062$$

$$(1) \quad 3.\overset{\curvearrowright}{1}\overset{\curvearrowright}{2} \quad \div \quad 0.\overset{\curvearrowright}{0}\overset{\curvearrowright}{0}\overset{\curvearrowright}{3}$$
$$= 3 \ 120 \quad \div \quad 3$$
$$= 1 \ 040$$

remove decimal
point from divisor:
multiply both
numbers by 1000

$$(2) \quad 3.12 \quad \div \quad 3000$$
$$= \overset{\curvearrowleft}{1}.\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{4} \quad \div \quad 1000$$
$$= 0.00104$$

divide by 3
then by 1000

$$(3) \quad 3.\overset{\curvearrowright}{1}\overset{\curvearrowright}{2} \quad \times \quad 3000$$
$$= 3 \ 120 \quad \times \quad 3$$
$$= 9 \ 360$$

multiply by 1000
then by 3

ADD and SUBTRACT

$$\begin{array}{r}
 \text{U t h th} \\
 \overset{8}{\cancel{9}}.\overset{1}{4}\overset{7}{\cancel{8}}\overset{1}{0} \\
 - 5.764 \\
 \hline
 3.716
 \end{array}$$

$$\begin{array}{r}
 \text{T U t h} \\
 8.37 \\
 9.60 \\
 + 5.49 \\
 \hline
 23.46 \\
 \hline
 \underset{2}{2}\underset{1}{3}\underset{1}{6}
 \end{array}$$

MULTIPLY and DIVIDE

$$\begin{array}{r}
 4.547 \\
 \times 6 \\
 \hline
 27.282 \\
 \hline
 \underset{3}{2}\underset{2}{7}\underset{4}{2}
 \end{array}$$

$$\begin{array}{r}
 04.547 \\
 6 \overline{) 27.282}
 \end{array}$$

STANDARD FORM (SCIENTIFIC NOTATION)

Used to write very large and very small numbers.

Form $a \times 10^n$

$1 \leq a < 10$

ie. between 1 and 10, excluding 10

n is an INTEGER

ie. ...-3,-2,-1,0,1,2,3...

$$257000 = 2.57 \times 10^5$$

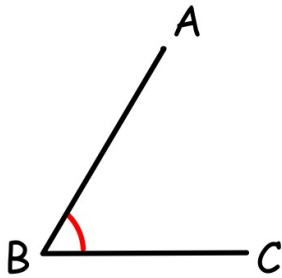
$$257000.$$

$$0.0000257 = 2.57 \times 10^{-5}$$

$$0.0000257$$

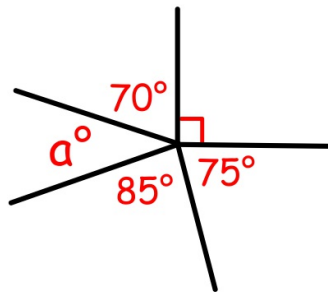
CHAPTER 3: ANGLES

NAMING ANGLES: 3 capital letters and an angle sign
the middle letter is the VERTEX.



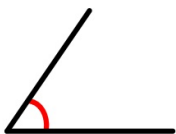
arms: **AB** and **BC**
vertex: **B**
angle: **$\angle ABC$**

COMPLETE TURN:
360° around a point

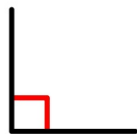


70	
85	
75	360
+ 90	- 320
320	40
	$\alpha = 40$

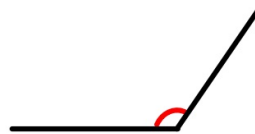
DESCRIPTIONS (TYPES):



acute angle:
between 0° and 90°



right angle:
90°

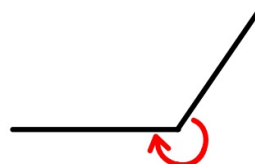
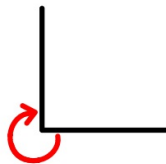
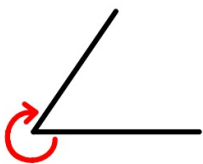


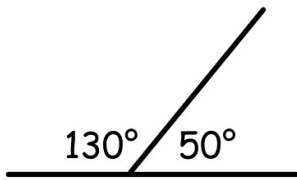
obtuse angle:
between 90° and 180°



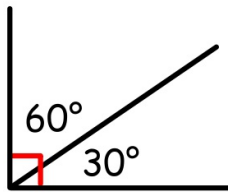
straight angle:
180°

Reflex angle: between 180° and 360°

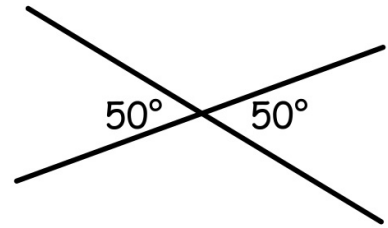




supplementary

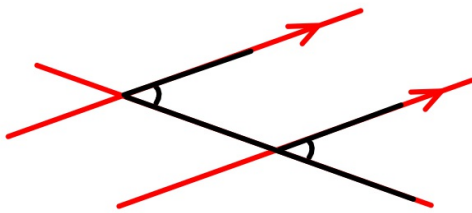


complementary

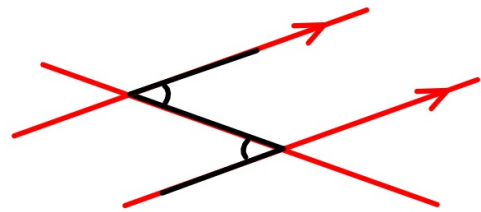


vertically opposite
(X shape)

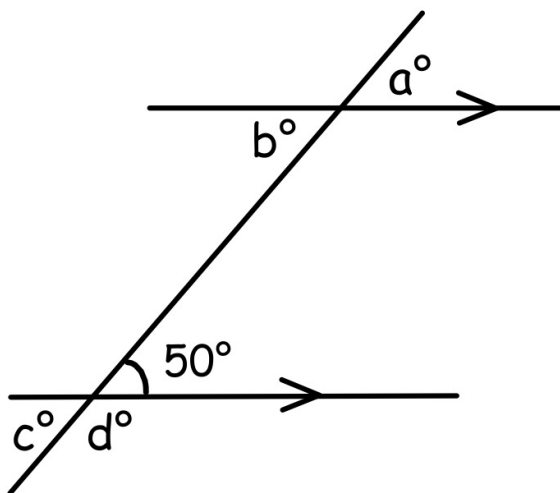
PARALLEL LINES:



corresponding angles equal
(F shape)



alternate angles equal
(Z shape)



$a = 50$ corresponding (F shape)

$b = 50$ alternate (Z shape)

$c = 50$ vertically opposite
(X shape)

$d = 130$ supplementary

	180
-	50
	<u>130</u>

CHAPTER 4: ALGEBRA

SHORT FORMS

multiplication is repeated addition

$$(i) \quad 7 + 7 + 7 + 7 + 7 + 7 = 6 \times 7$$

$$(ii) \quad a + a + a + a + a + a = 6 \times a = 6a$$

$$(iii) \quad a^2 + a^2 + a^2 + a^2 + a^2 + a^2 = 6 \times a^2 = 6a^2$$

a power is repeated multiplication

$$(i) \quad 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6$$

$$(ii) \quad a \times a \times a \times a \times a \times a = a^6$$

"Like Terms" can be added/subtracted.
(same letter combinations)

$$\begin{array}{lll} (1) \quad 5a - 2b + 3a & (2) \quad 5ab - 2ba + ab & (3) \quad 7a^2 - 4a - 6a^2 \\ = 5a + 3a - 2b & = 5ab - 2ab + 1ab & = 7a^2 - 6a^2 - 4a \\ = 8a - 2b & = 4ab & = a^2 - 4a \end{array}$$

Multiplication combines letters and numbers.

$$\begin{array}{ll} (4) \quad 2a^2 \times 4ab & (5) \quad 3c^2 \times 2c - 5 \times c \times c \times d \\ = 2 \times a \times a \times 4 \times a \times b & = 6c^3 - 5c^2d \\ = 2 \times 4 \times a \times a \times a \times b & \\ = 8a^3b & \end{array}$$

BRACKET BREAKING

$$a \times (b + c) = a \times b + a \times c$$

multiply each term in the brackets

$$\begin{aligned} (1) \quad & 6(2w - 3y) \\ & = 12w - 18y \end{aligned}$$

$$\begin{aligned} (2) \quad & 8p(2p + r) \\ & = 16p^2 + 8pr \end{aligned}$$

remove brackets then simplify:

$$\begin{aligned} (3) \quad & 2(3p + 5) - 7 \\ & = 6p + 10 - 7 \\ & = 6p + 3 \end{aligned}$$

$$\begin{aligned} (4) \quad & n(4n - 3) + n^2 \\ & = 4n^2 - 3n + n^2 \\ & = 5n^2 - 3n \end{aligned}$$

FACTORISATION

$$a \times b + a \times c = a \times (b + c)$$

To FULLY factorise **a** is the Highest Common Factor

$$\begin{aligned} (1) \quad & 12w - 18y \\ & = 6(2w - 3y) \end{aligned}$$

$$\begin{aligned} (2) \quad & 4n^2 + 5mn - n \\ & = n(4n + 5m - 1) \end{aligned}$$

$$\begin{aligned} (3) \quad & 16p^2 + 8pr \\ & = 8p(2p + r) \end{aligned}$$

$$\begin{aligned} (4) \quad & 1.4 \times 3.6 - 1.4 \times 1.6 \\ & = 1.4 \times (3.6 - 1.6) \\ & = 1.4 \times 2 \\ & = 2.8 \end{aligned}$$

CHAPTER 5: INFORMATION HANDLING 1

AVERAGES:

$$\text{mean} = \frac{\text{total of all results}}{\text{number of results}}$$

median: the middle result of the ordered results

(if an even number of results - find the mean of the middle two results)

mode: the most common result

SPREAD:

range = highest result - lowest result

19,14,15,19,16,22,16,20,24,14,25,18,27,12,16

$$\text{mean} = \frac{277}{15} = 18.466\dots = 18.5$$

15 ordered results: 8th result in the middle

12,14,14,15,16,16,16,18,19,19,20,22,24,25,27

median = 18

mode = 16

range = 27 - 12 = 15

median for even number of results:

16 ordered results: 8th/9th results in the middle

12,14,14,15,16,16,16,18,19,19,20,22,24,25,27,30

$$\text{median} = (18+19) \div 2 = 18.5$$

MEAN PROBLEMS

Total = mean \times number of results

- (1) 15 girls, mean height 1.2m
10 boys, mean height 1.4m

$$1.2 \text{ m} \times 15 = 18 \text{ m}$$

$$1.4 \text{ m} \times 10 = 14 \text{ m}$$

Find the mean height of
the group of 25 children ?

$$\text{total height} \quad \underline{32 \text{ m}}$$

$$\text{mean} = 32 \div 25 = \underline{\underline{1.28 \text{ m}}}$$

- (2) 4 boys have mean weight 60 kg
When John is included the
mean weight is 62 kg
Find the weight of John.

$$60 \text{ Kg} \times 4 = 240 \text{ Kg}$$

$$62 \text{ Kg} \times 5 = 310 \text{ Kg}$$

$$310 - 240 = \underline{\underline{70 \text{ Kg}}}$$

FREQUENCY TABLES

If there is a lot of data the information is easier to present in a frequency distribution table.

TEST SCORES: 1,1,2,2,2,2,2,3,3,3,3,3,3,4,4,4,5,5

result	tally	frequency
1		3
2		5
3		7
4		3
5		2

MEAN FROM A FREQUENCY TABLE:

A result x frequency column is included.

$$\text{mean} = \frac{\text{total of: result} \times \text{frequency}}{\text{total frequency}}$$

result	frequency	result x frequency
1	3	3
2	5	10
3	7	21
4	3	12
5	2	10
TOTALS	20	56

mean = $56 \div 20$
= 2.8

GROUPED FREQUENCY TABLES

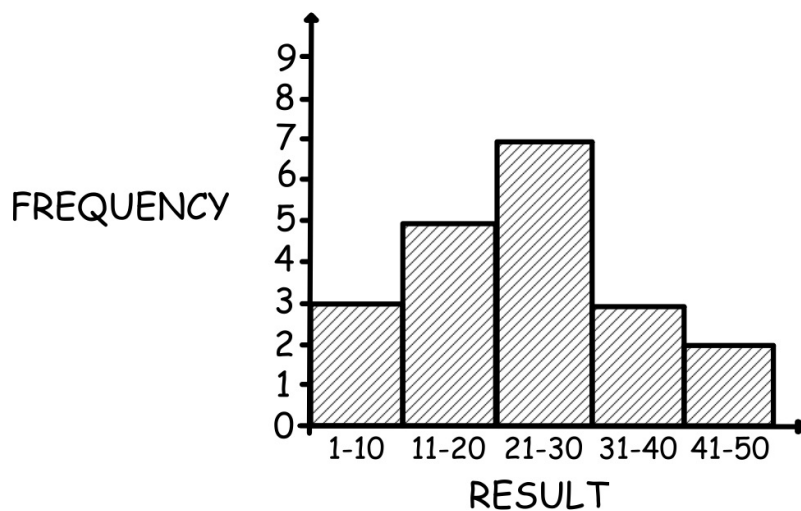
Data can be grouped into class intervals

TEST SCORES: 3,5,8 , 11,13,17,17,19,
22,23,24,25,26,28,29,
33,38,39 , 44,47

result	tally	frequency
1 - 10		3
11 - 20		5
21 - 30		7
31 - 40		3
41 - 50		2

The results can be shown on a frequency diagram.

The **MODAL CLASS** has the highest frequency.



MODAL CLASS is 21-30

CHAPTER 6: TIME and SPEED

CLOCK TIME:

12 hour	6.25 am	6.25 pm	noon	midnight
24 hour	06 25	18 25	12 00	00 00 24 00

Note: 6.25 am is 6hr 25min into the day

6.25 hr is 6hr 15min

(6¹/₄ hr)

TIME DIFFERENCES

(1) 8.25 am to 4.10 pm

08 25 to 09 00	35 min
09 00 to 16 10	<u>7 hr 10 min</u>
	7 hr 45 min

(2) 10.35 pm to 2.20 am

22 35 to 24 00	1 hr 25 min
00 00 to 02 20	<u>2 hr 20 min</u>
	3 hr 45 min

SPEED



$$S = \frac{D}{T}$$

minutes \div 60 = hours



$$D = ST$$



$$T = \frac{D}{S}$$

hours \times 60 = minutes

- (1) Travel 240 miles in 4 hours 10 min.
Find the average speed.

$$S = \frac{D}{T}$$

$$\begin{aligned} &= 240 \div 4.166666666 \\ &= 57.6 \text{ mph} \end{aligned}$$

$10 \div 60 + 4$

$240 \div \boxed{\text{ANS}} =$

- (2) How far is travelled at 36 mph for 4 hours 35 min. ?

$$D = ST$$

$$\begin{aligned} &= 36 \times 4.583333333 \\ &= 165 \text{ miles} \end{aligned}$$

$35 \div 60 + 4$

- (3) How long does it take to travel 100 miles at 24 mph ?

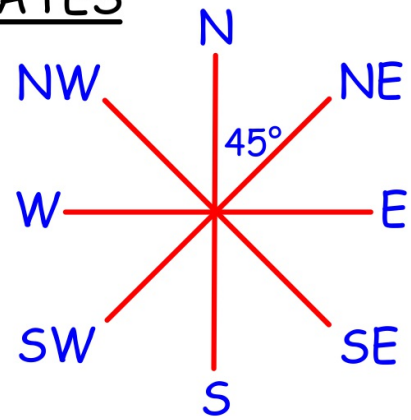
$$T = \frac{D}{S}$$

$$\begin{aligned} &= 100 \div 24 \\ &= 4.166666666 \text{ hours} \\ &= 4 \text{ hours } 10 \text{ min} \end{aligned}$$

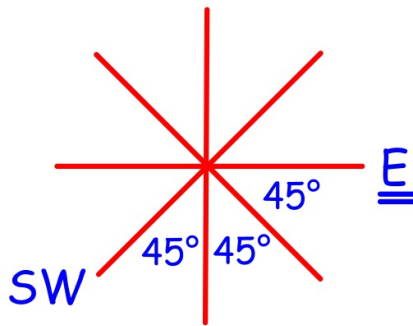
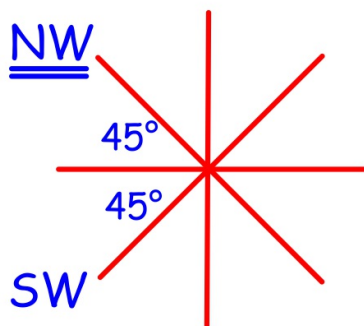
$0.1666... \times 60 = 10$

CHAPTER 7: SCALES and COORDINATES

8 POINT COMPASS

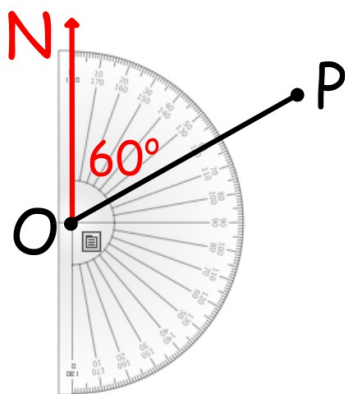


Facing SW, what direction do I face after turning:
 (a) clockwise 90° (b) anticlockwise 135° ?

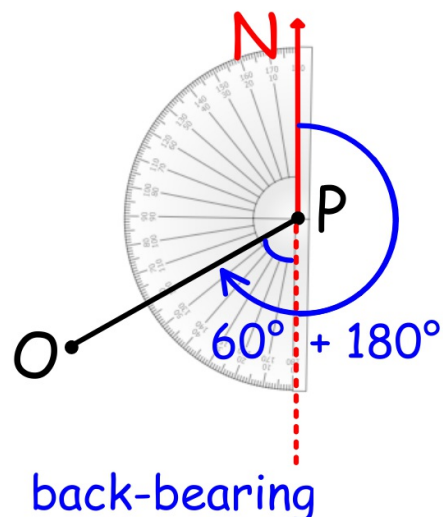


THREE FIGURE BEARINGS

The angle of turning clockwise from North.



P from O is 060°



O from P is 240°

The bearing and back-bearing differ by 180°

SCALE AS RATIO: REPRESENTATIVE FRACTION

$$1 \text{ cm} = 2 \text{ km}$$

$$\begin{aligned} & 1 \text{ cm} : 2 \text{ km} \\ &= 1 \text{ cm} : 2000 \text{ m} \\ &= 1 \text{ cm} : 200000 \text{ cm} \\ &= 1 : 200000 \end{aligned}$$

(i) $1 : 200000$

map distance 7.2 cm

real distance: ?

$$\begin{aligned} & 7.2 \text{ cm} \times 200000 \\ &= 1440000 \text{ cm} \\ &= 14400 \text{ m} \\ &= 14.4 \text{ km} \end{aligned}$$

(ii) $1 : 50000$

real distance 3.4 km

map distance: ?

$$\begin{aligned} & 3.4 \text{ km} \div 50000 \\ &= 3400 \text{ m} \div 50000 \\ &= 340000 \text{ cm} \div 50000 \\ &= 34 \text{ cm} \div 5 \\ &= 6.8 \text{ cm} \end{aligned}$$

(iii) real distance 7 m

map distance 3.5 cm

representative fraction: ?

$$\begin{aligned} & 3.5 \text{ cm} : 7 \text{ m} \\ &= 3.5 \text{ cm} : 700 \text{ cm} \\ &= 1 : 200 \end{aligned}$$

by $700 \div 3.5$

SCALE DRAWING

Suitable scale for a big diagram.

Neat and accurate, labelled with lengths and angles

Write the measurements made and all the calculations

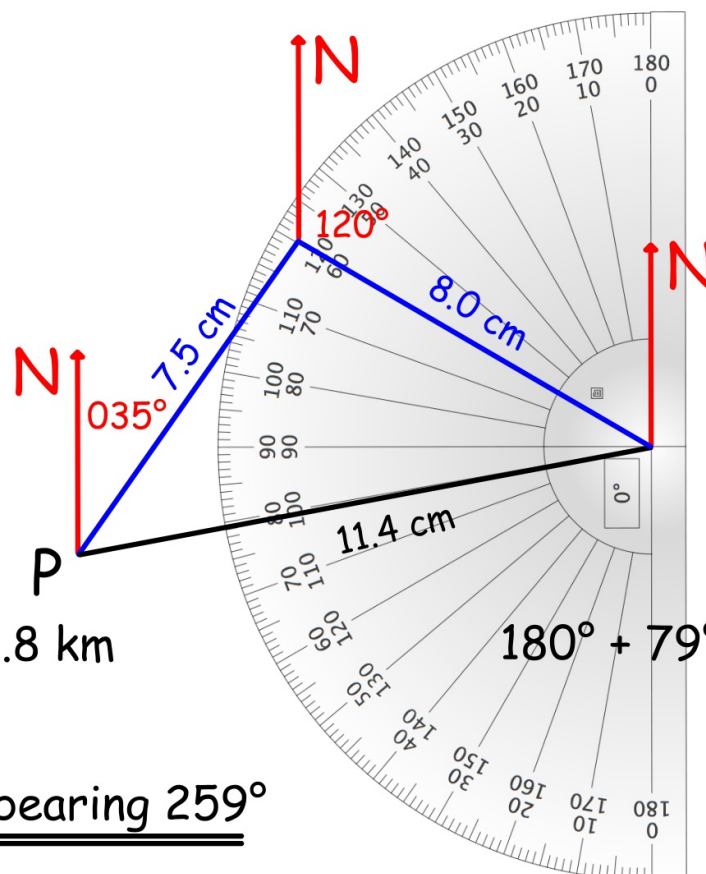
A ship leaves port and sails for 15 km on bearing 035° and then for 16 km on bearing 120°.

Find the distance and bearing to return to port.

$$1 \text{ cm} = 2 \text{ km}$$

$$15.0 \div 2 = 7.5 \text{ cm}$$

$$16.0 \div 2 = 8.0 \text{ cm}$$



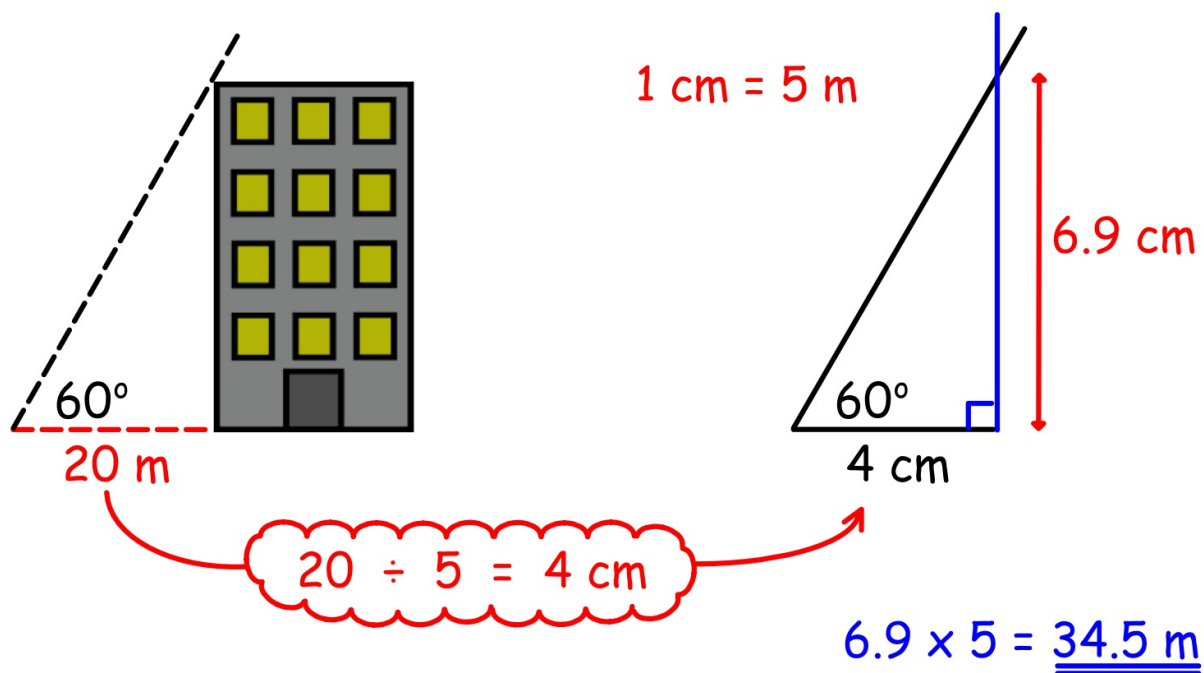
$$11.4 \times 2 = 22.8 \text{ km}$$

$$180^\circ + 79^\circ = 259^\circ$$

22.8 km on bearing 259°

SCALE DRAWING

Find the height of the building.



COORDINATES

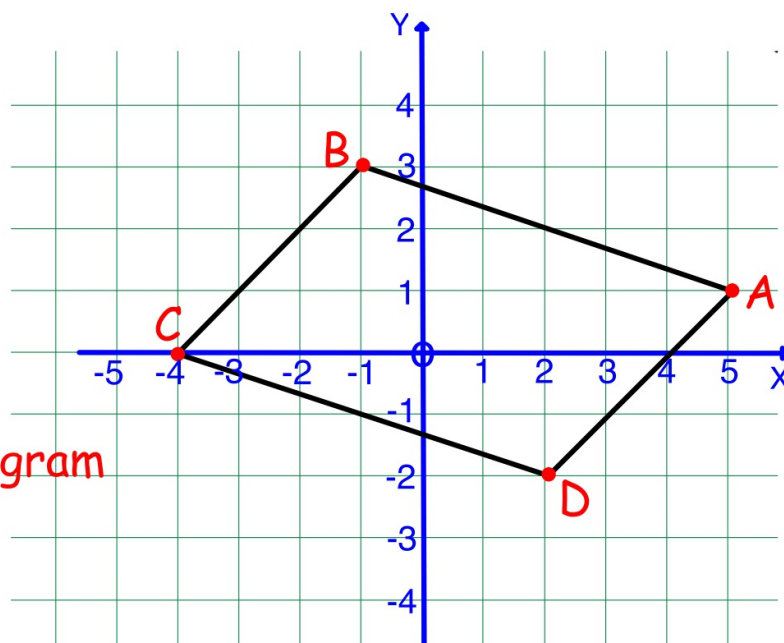
A (5,1) x-coordinate is 5
y-coordinate is 1

B (-1,3)

C (-4,0)

D (2,-2)

ABCD is a parallelogram

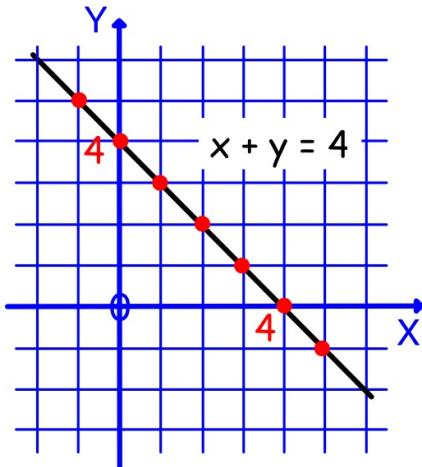


LINES

The equation gives a rule connecting the x and y coordinates of any point on the line.

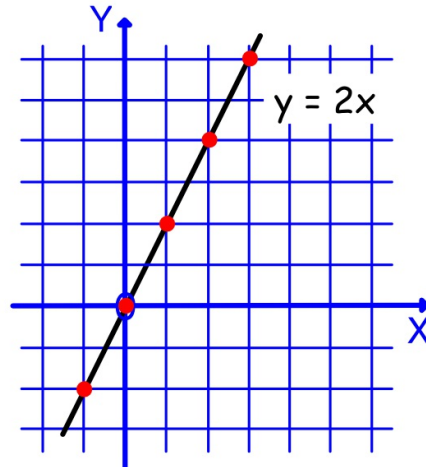
$$x + y = 4$$

x	-1	0	1	2	3	4	5
y	5	4	3	2	1	0	-1



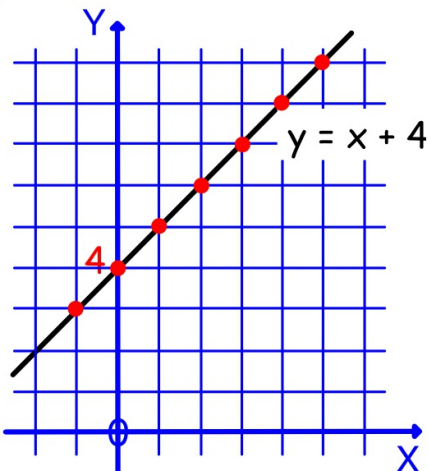
$$y = 2x$$

x	-1	0	1	2	3
y	-2	0	2	4	6



$$y = x + 4$$

x	-1	0	1	2	3	4	5
y	3	4	5	6	7	8	9



VERTICAL and HORIZONTAL lines

