CHAPTER 8: EQUATIONS and INEQUATIONS

SIMPLE EQUATIONS: 'cover-up' type

isolate the term with the letter

(1)
$$2x - 3 = 11$$
$$2x = 14$$
$$x = 7$$

(2)
$$21 = 6 + 5a$$
$$5a = 15$$
$$a = 3$$

(3)
$$17 - 3w = 5$$
$$3w = 12$$
$$w = 4$$

(4)
$$\frac{1}{2}h - 3 = 2$$
$$\frac{1}{2}h = 5$$
$$h = 10$$

LETTERS ON BOTH SIDES - get letters on one side

(1)
$$5x - 2 = 2x + 10$$ subtract 2x from each side
$$3x - 2 = 10$$ add 2 to each side
$$3x = 12$$ divide each side by 3
$$x = 4$$

(2)
$$2 + 3n = 5 - 4n$$ add 4n to each side
$$2 + 7n = 5$$ subtract 2 from each side
$$7n = 3$$ divide each side by 7
$$n = \frac{3}{7}$$

EQUATIONS and negatives

(1) $17 - 2y = 3$ — subtract 17 from each side

$-2y = -14$ — divide each side by -2

$y = 7$

(2) $8 + 2n = 6 - 3n$ — add 3n to each side

$8 + 5n = 6$ — subtract 8 from each side

$5n = -2$ — divide each side by 5

$n = -{}^{2}/_{5}$

EQUATIONS with fractions

first remove fractions: multiply by the denominator.

(1) $\frac{x}{2} = 5$

multiply by 2

$x = 10$

(2) $\frac{x - 3}{4} = -2$

multiply by 4

$x - 3 = -8$

$x = -5$

NOTE: same equations

${}^{1}/_{2}\, x = 5$

${}^{1}/_{4}\,(x - 3) = -2$

BRACKET BREAKING

$$a \times (b + c) = a \times b + a \times c$$

(1) $3p(2p + r)$

$= 6p^2 + 3pr$

(2) $2a(3a - b + 5)$

$= 6a^2 - 2ab + 10a$

signs change when multiplying by a negative term:

(3) $-3(2w - 3y)$

$= -6w + 9y$

(4) $-n(4n + 5m)$

$= -4n^2 - 5mn$

EXPRESSIONS: remove brackets then simplify

(1) $2a + 3a(2 - 3a)$

$= 2a + 6a - 9a^2$

$= 8a - 9a^2$

(2) $5 - 3(2a - 3)$

$= 5 - 6a + 9$

$= 14 - 6a$

no sign change

sign change

EQUATIONS: remove brackets then solve

(1) $2(w + 6) = 5(w - 3)$

$2w + 12 = 5w - 15$

$12 = 3w - 15$

$27 = 3w$

$w = 9$

(2) $3y = 14 - 2(y - 3)$

$3y = 14 - 2y + 6$

$3y = 20 - 2y$

$5y = 20$

$y = 4$

INEQUALITIES (INEQUATIONS)

$>$	greater than	eg. $7 > 3$
$\geq$	greater than or equal to	
$<$	less than	eg. $3 < 7$
$\leq$	less than or equal to	

solve for $x = 1, 2, 3, 4, 5, 6$

(1) $x > 4$
$x = 5,6$

(2) $x \geq 4$
$x = 4,5,6$

(3) $x < 4$
$x = 1,2,3$

follow the same rules as equations

(1)
$$5x - 4 < 6$$
$$5x < 10$$
$$x < 2$$

(2)
$$2x + 7 \leq 1$$
$$2x \leq -6$$
$$x \leq -3$$

(3)
$$5x < 3x + 10$$
$$2x < 10$$
$$x < 5$$

(4)
$$2x \geq 20 - 3x$$
$$5x \geq 20$$
$$x \geq 4$$

$\times$ or $\div$ by a negative: reverse the inequality sign

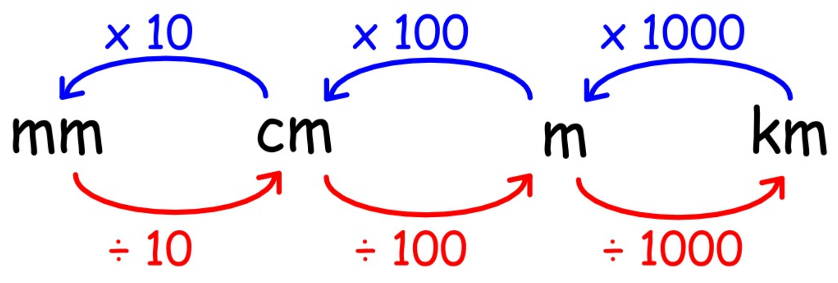
(i)
$$5x \geq 30$$
$$x \geq 6$$
no sign change

(ii)
$$-5x \geq 30$$
$$x \leq -6$$
sign change

CHAPTER 9: LENGTH and PYTHAGORAS' THEOREM

LENGTH UNITS

multiply going to smaller units



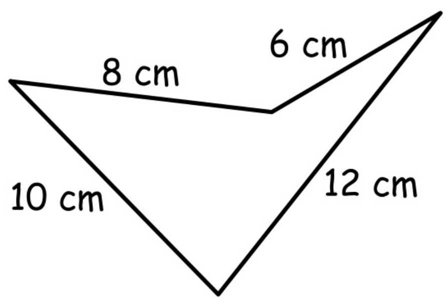
divide going to larger units

18 mm ÷ 10 = 1.8 cm	0.7 m × 100 = 70 cm
3.4 m × 100 = 340 cm	520 cm ÷ 100 = 5.20 m
40 m ÷ 1000 = 0.040 km	0.2 km × 1000 = 200 m

PERIMETER

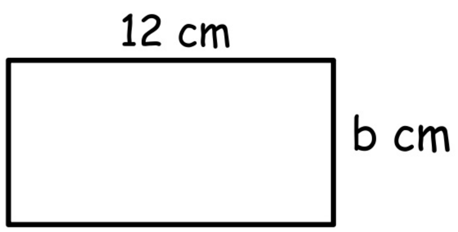
The total distance around the outside edge.

(1)



P = 12 + 10 + 8 + 6
= 36 cm

(2)

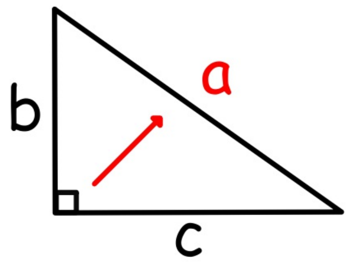


rectangle perimeter 40 cm

P = 12 + 12 + b + b
40 = 24 + 2b
2b = 16
b = 8

PYTHAGORAS' THEOREM

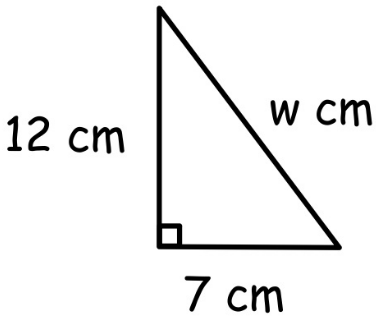
For right-angled triangles only:



$$a^2 = b^2 + c^2$$

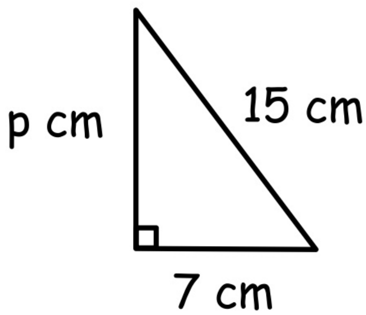
HYPOTENUSE (largest side) is opposite the 90° angle.

BIGGEST SIDE



$$w^2 = 12^2 + 7^2$$
$$= 144 + 49$$
$$= 193$$
$$w = \sqrt{193}$$
$$= 13.892...$$
$$w \approx 13.9$$

SMALLER SIDE



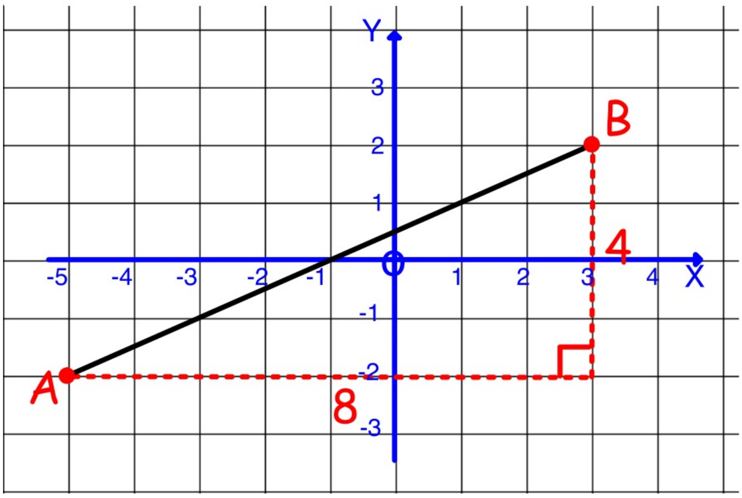
$$p^2 = 15^2 - 7^2$$
$$= 225 - 49$$
$$= 176$$
$$p = \sqrt{176}$$
$$= 13.266...$$
$$p \approx 13.3$$

DISTANCE BETWEEN TWO POINTS

Plot the points.

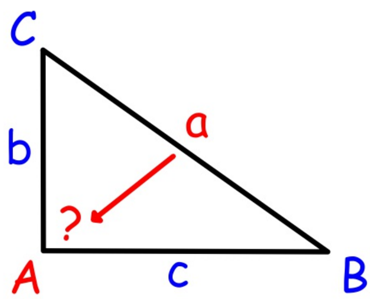
Construct the right-angled triangle around them.

A (-5,-2) and B (3,2)



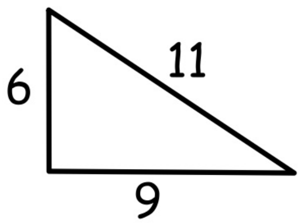
$$AB^2 = 8^2 + 4^2$$
$$= 64 + 16$$
$$= 80$$
$$AB = \sqrt{80}$$
$$= 8.944...$$
$$AB \approx 8.9 \text{ units}$$

CONVERSE OF PYTH. THM.



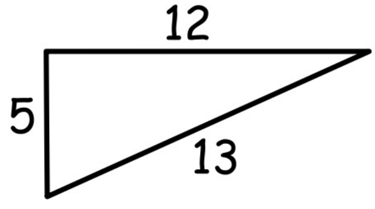
if $a^2 = b^2 + c^2$

then ΔABC is right-angled



$6^2 + 9^2 \neq 11^2$

Δ is not right-angled

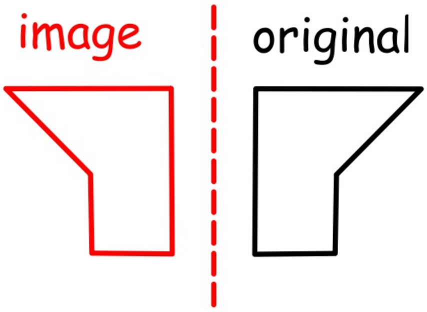


$5^2 + 12^2 = 13^2$

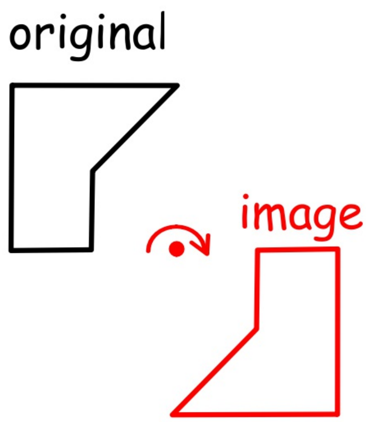
Δ is right-angled

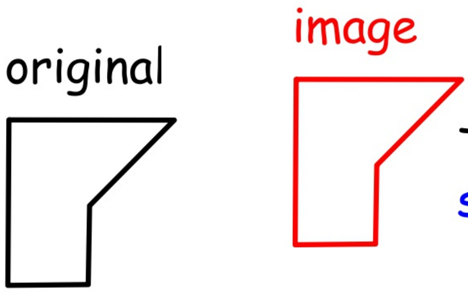
CHAPTER 10: TRANSFORMATIONS

REFLECTION
folding over a line



ROTATION
turning about a point





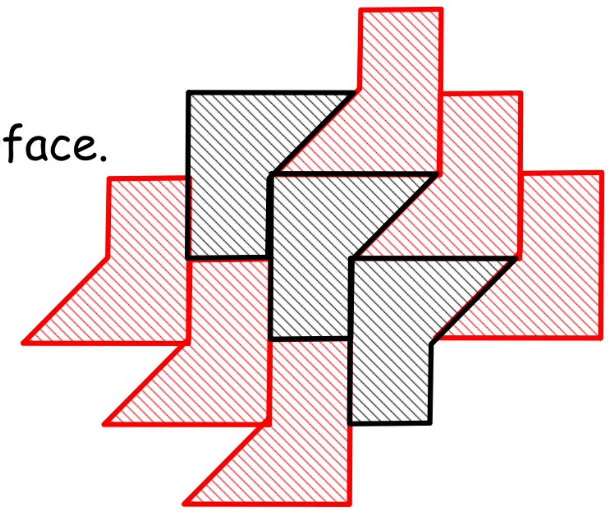
TRANSLATION
sliding movement

TILINGS

CONGRUENT tiles cover a surface.
(identical)

No gaps or overlap.

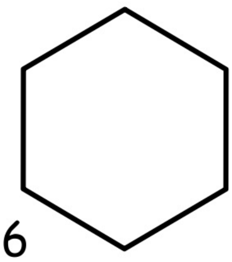
Can extend in any direction.



ORDER OF ROTATIONAL SYMMETRY

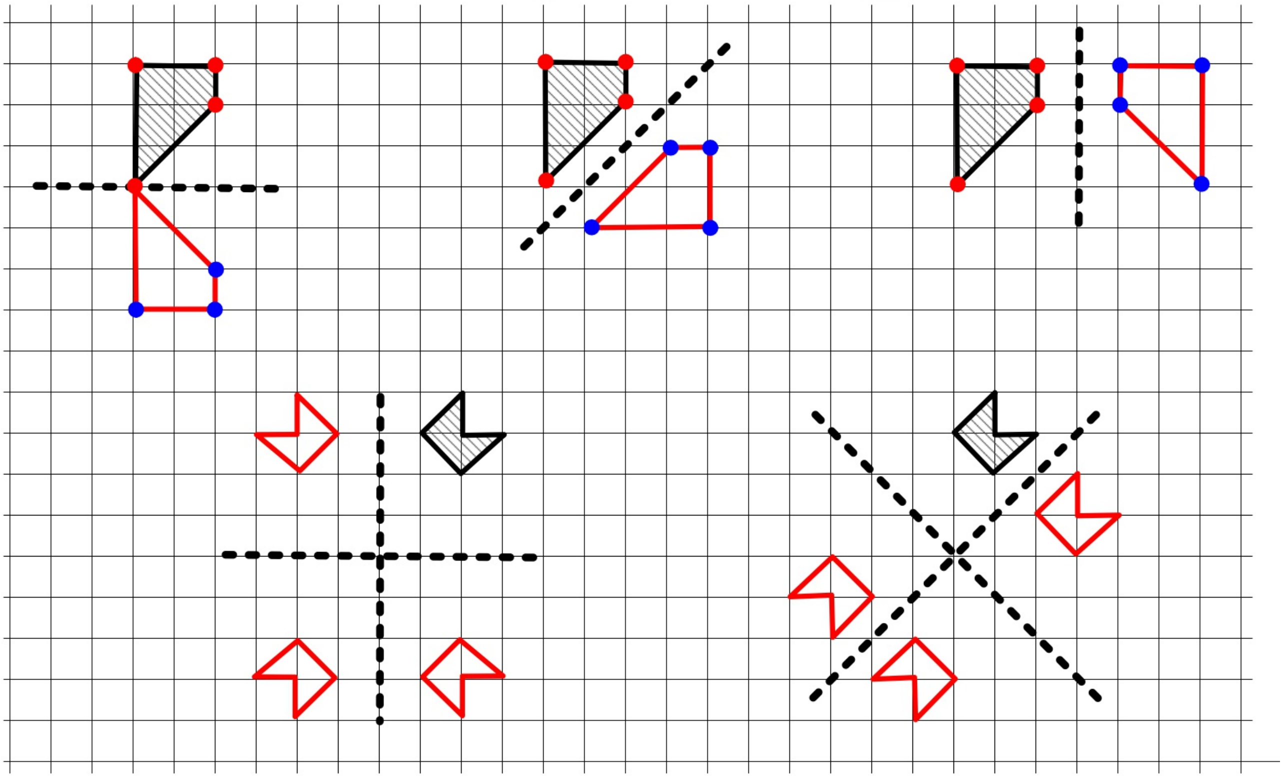
The number of times a shape fits itself under one turn.

order 6

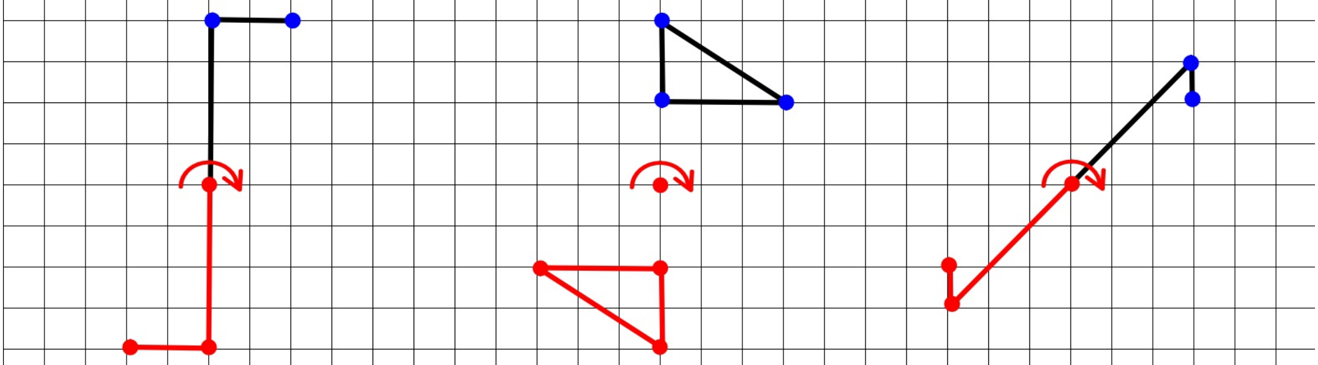


Line symmetry

Reflect the corners and join for the image.



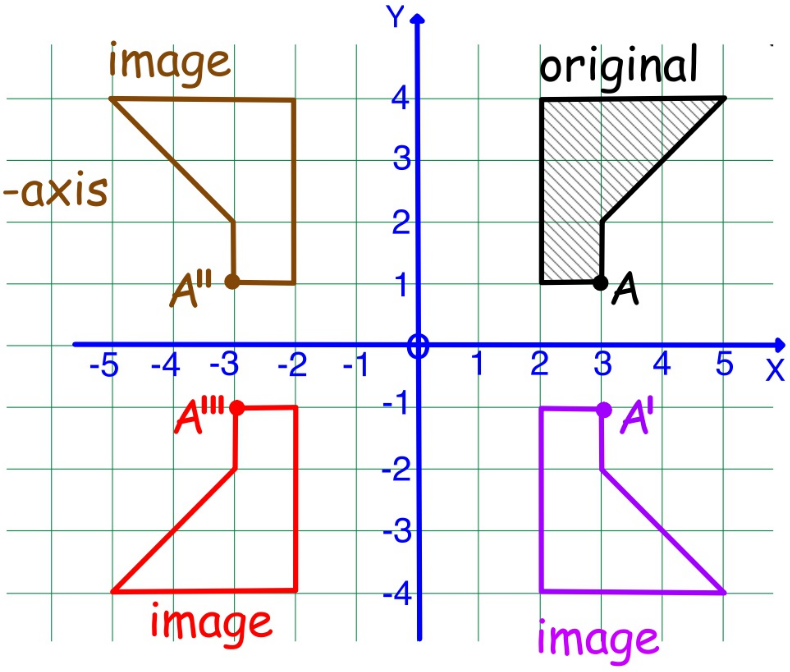
Half-turn symmetry



Quarter-turn symmetry

COORDINATES

reflection in the Y-axis

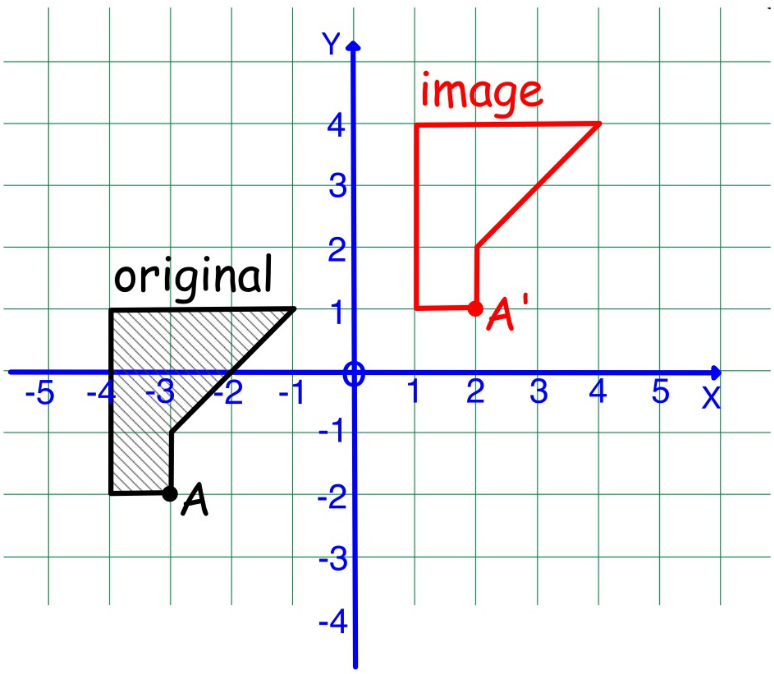


half-turn about O

reflection in the X-axis

image under A(-3,-2) ⟶ A'(2,1)

all points move
5 right , 3 up



ENLARGEMENT and REDUCTION

Angles are unchanged.

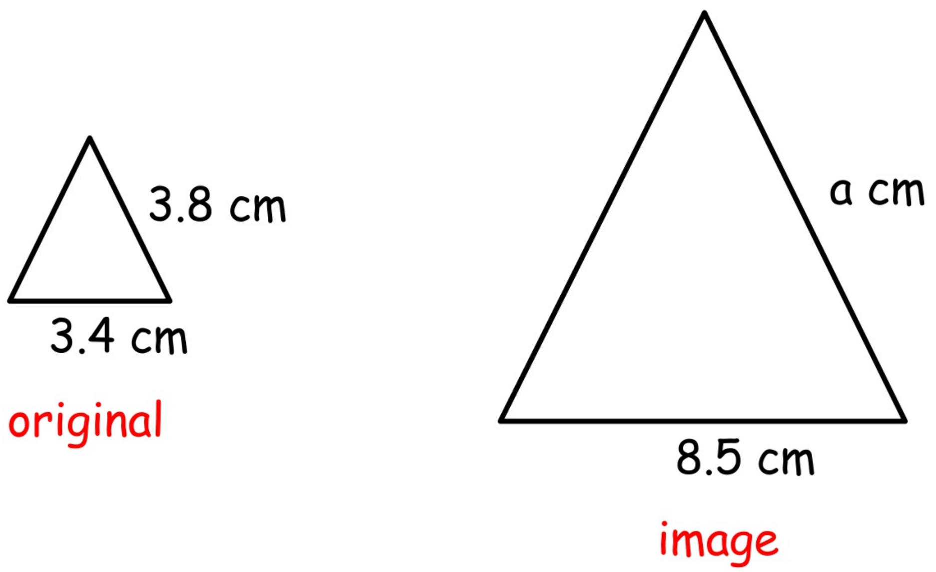
Sides are enlarged/reduced by a SCALE FACTOR.

ENLARGEMENT: $SF > 1$

REDUCTION: $0 < SF < 1$

The shape with the dimension to be found is the image.

$$\text{Scale Factor} = \frac{\text{image size}}{\text{original size}}$$



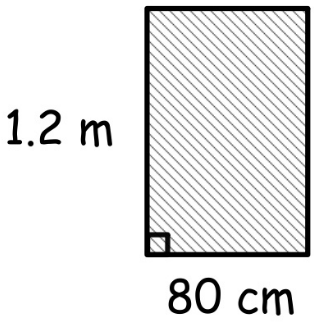
$$SF = \frac{8.5}{3.4} = 2.5$$

$$a = 3.8 \times 2.5 = 9.5$$

CHAPTER 11: AREA

RECTANGLE $A = lb$

match length units to required area units

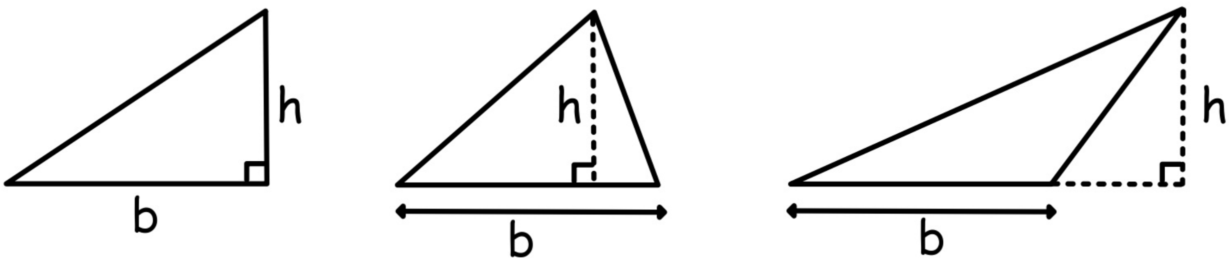


$A = lb$
$= 1.2 \times 0.8$
$= 0.96\ m^2$

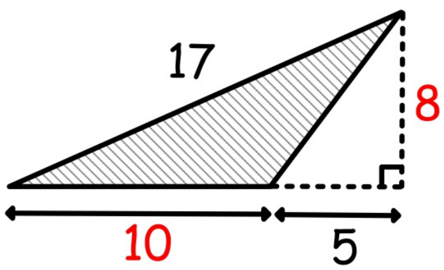
$A = lb$
$= 120 \times 80$
$= 9600\ cm^2$

TRIANGLES $A = \frac{1}{2} bh$

The height and the base are at 90°.
(altitude) (perpendicular)



use base and height - ignore extra numbers

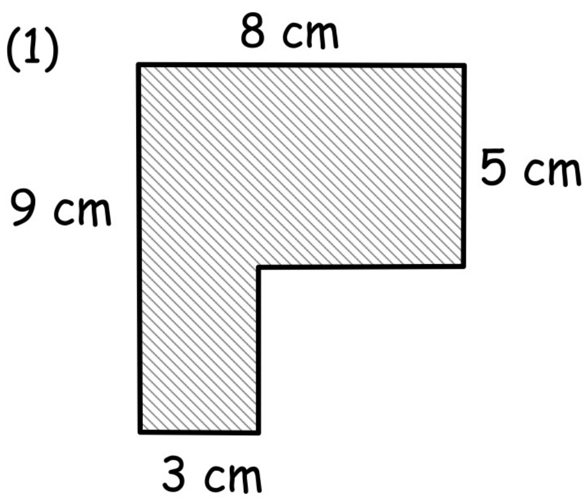


$A = \frac{1}{2} bh$
$= 10 \times 8 \div 2$
$= 40\ units^2$

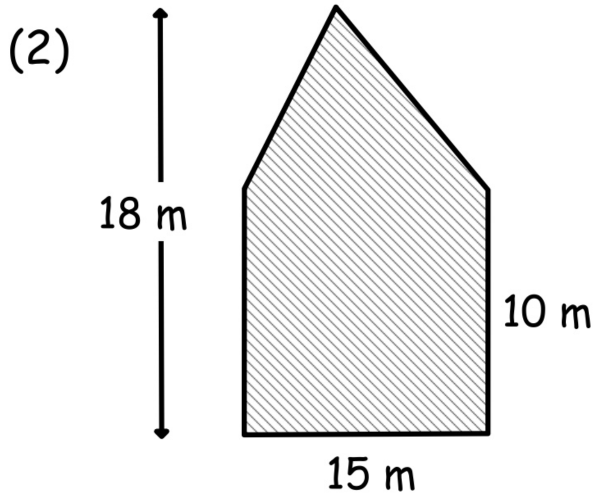
COMPOSITE SHAPES

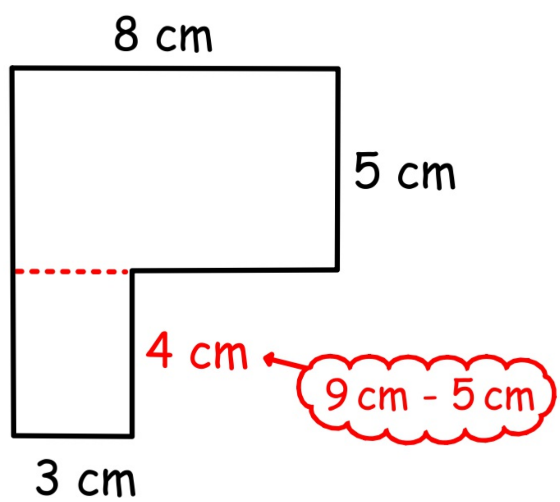
Formed from rectangles and triangles

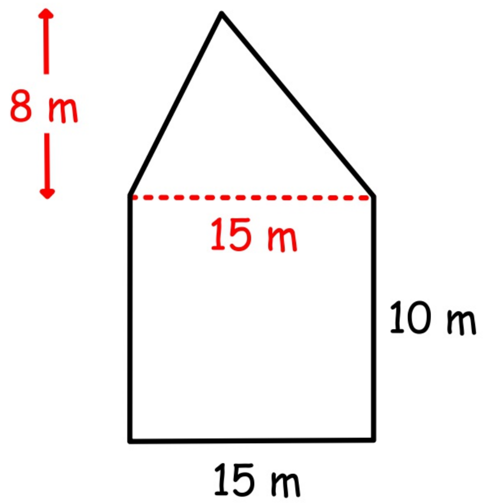
(1)



(2)







$A = lb$
$= 8 \times 5$
$= 40\ cm^2$

$A = lb$
$= 4 \times 3$
$= 12\ cm^2$

$40\ cm^2 + 12\ cm^2$
$= 52\ cm^2$

$A = lb$
$= 15 \times 10$
$= 150\ m^2$

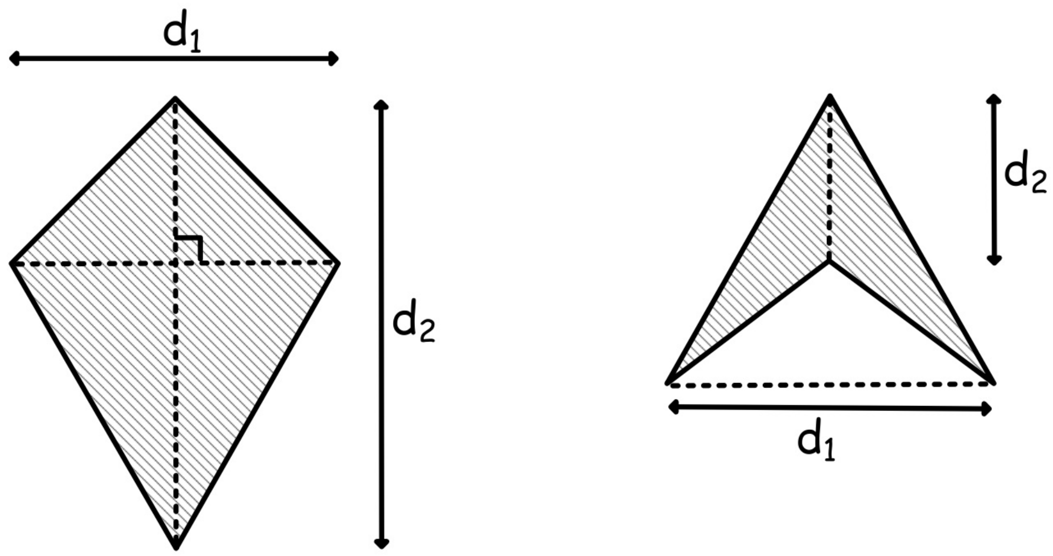
$A = {}^1/_2\ bh$
$= 15 \times 8 \div 2$
$= 60\ m^2$

$150\ m^2 + 60\ m^2$
$= 210\ m^2$

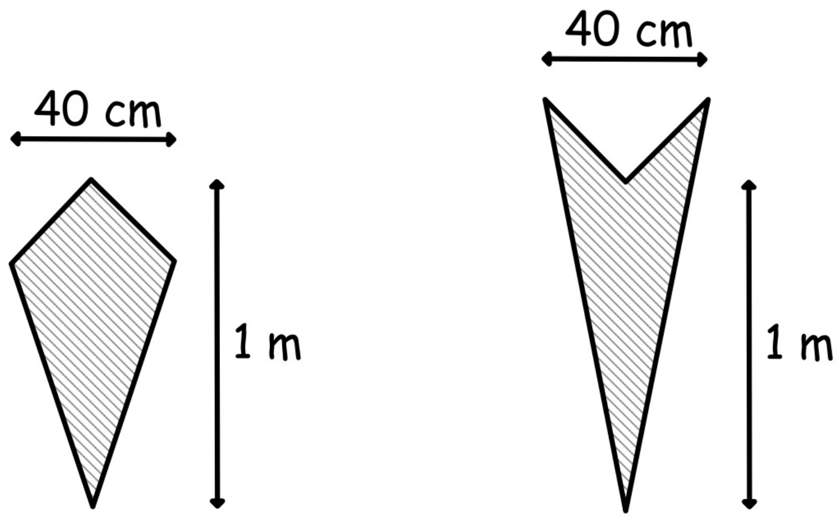
KITE and RHOMBUS

A = $^1/_2$ the product of the diagonals

$A = {}^1/_2\, d_1d_2$



Ensure the units match.



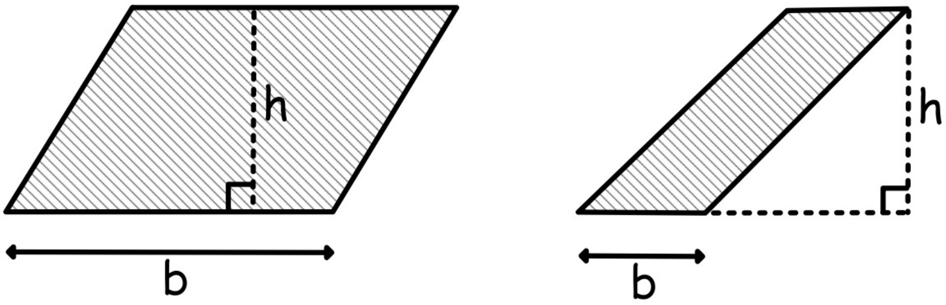
$A = {}^1/_2\, d_1d_2$
$= 40 \times 100 \div 2$
$= \underline{\underline{2000\ cm^2}}$

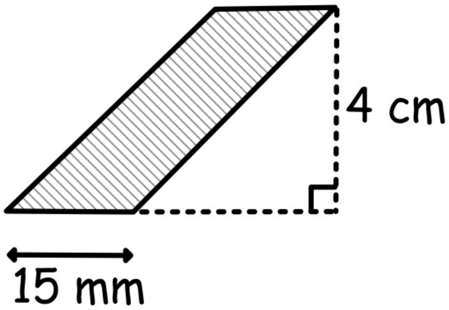
or

$A = {}^1/_2\, d_1d_2$
$= 0.4 \times 1 \div 2$
$= \underline{\underline{0.2\ m^2}}$

PARALLELOGRAM

$A = bh$





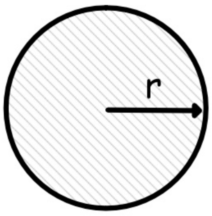
$A = bh$
$= 1.5 \times 4$
$= \underline{\underline{6\ cm^2}}$

or

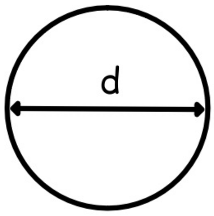
$A = bh$
$= 15 \times 40$
$= \underline{\underline{600\ mm^2}}$

CIRCLES

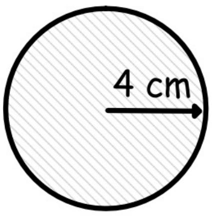
AREA: $A = \pi r^2$



CIRCUMFERENCE: $C = \pi d$



Remember $r = {}^1/_2\, d$ and $\pi \approx 3.14$

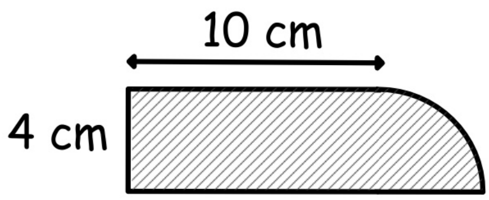


$A = \pi r^2$
$= 3.14 \times 4 \times 4$
$= 50.24\ cm^2$

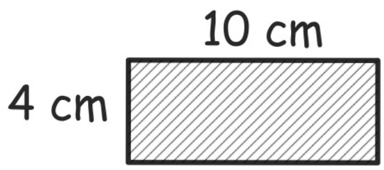
$C = \pi d$
$= 3.14 \times 8$
$= 25.12\ cm$

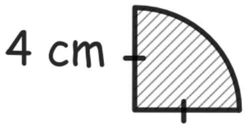
COMPOSITE SHAPES

Identify the rectangle and circle parts.



AREA:





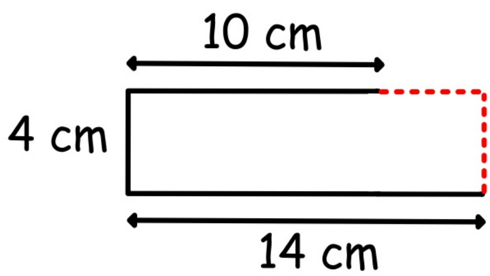
$A = lb$
$= 4 \times 10$
$= 40\ cm^2$

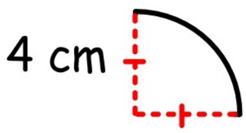
$A = \pi r^2$
$= 3.14 \times 4 \times 4$
$= 50.24\ cm^2$

$A = 50.24 \div 4$
$= 12.56\ cm^2$

Total Area = 12.56 + 40 = $52.56\ cm^2$

PERIMETER:





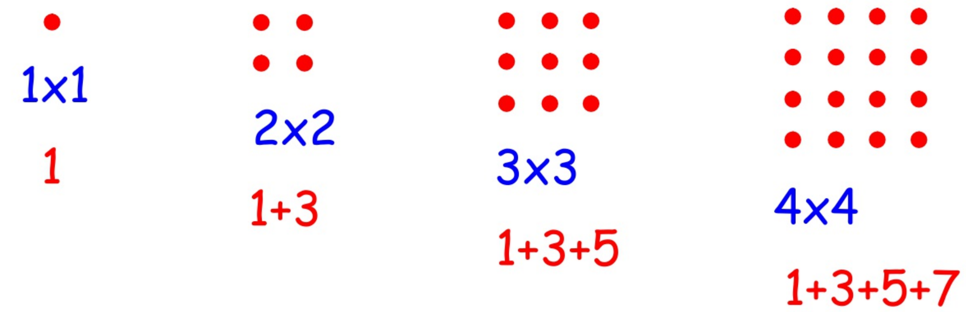
$C = \pi d$
$= 3.14 \times 8$
$= 25.12\ cm$

$25.12 \div 4$
$= 6.28\ cm$

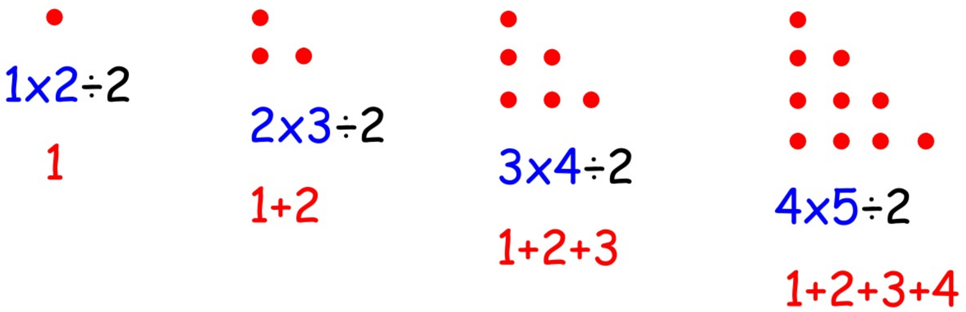
Perimeter = 4 + 10 + 14 + 6.28 = 34.28 cm

CHAPTER 12: SEQUENCES

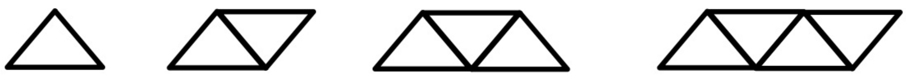
square numbers   0, 1, 4, 9, 16, 25 ...



triangular numbers   0, 1, 3, 6, 10, 15 ...



GENERALISE



find (a) a formula for the number of matches.

(b) the number of matches for 10 triangles.

(c) the number of triangles for 51 matches.

(a)

triangles	1	2	3	4	5	T
matches	3	5	7	9	11	2T + 1

add 2

Formula: $M = 2T + 1$

(b) $M = 2 \times 10 + 1 = 21$

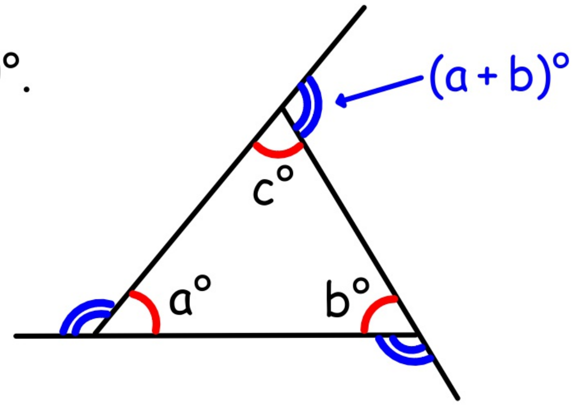
(c) $2T + 1 = 51$

$2T = 50$

$T = 25$

CHAPTER 13: TRIANGLES

INTERIOR angles add up to 180°.

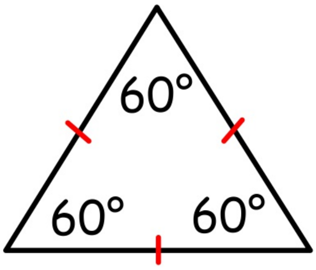


interior angles

exterior angles

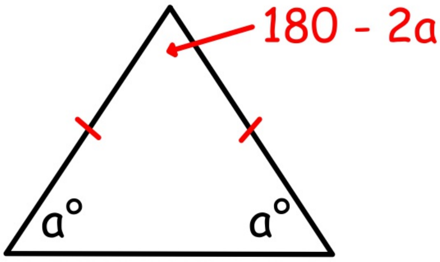
EQUILATERAL TRIANGLES

Three equal sides,
three equal angles of 60°.

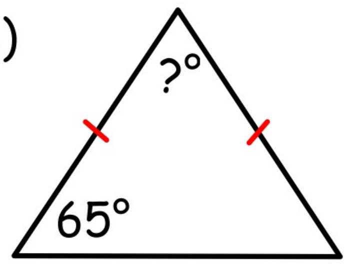


ISOSCELES TRIANGLES

Two equal sides, two equal angles.

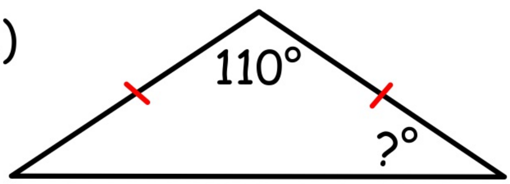


(1)



$$\begin{array}{r} 65 \\ +\ 65 \\ \hline 130 \end{array} \qquad \begin{array}{r} 180 \\ -\ 130 \\ \hline 50 \end{array}$$

(2)



$$\begin{array}{r} 180 \\ -\ 110 \\ \hline 70 \end{array} \qquad 2\overline{)\,7\,^{1}0} = 35$$

CHAPTER 14: RATIO and PROPORTION

RATIO   compare quantities - no units, fully simplify

(1)   6 kg : 900 g   same units

= 6000 g : 900 g   divide by 100

= 60 : 9   divide by 3

= 20 : 3

(2) Mix yellow and blue paint in the ratio 2 : 3
How much blue paint for 8 tins of yellow ?

Y : B   yellow tins: 8 ÷ 2 = 4

= 2 : 3   multiply by 4

= 8 : 12   12 tins of blue

SHARING

Tim and Tom buy 60 chocolates.
Tim contributes £3 and Tom £2.
How many chocolates should each get ?

share 60 chocs in ratio 3:2

number of shares   3 + 2 = 5 shares
one share   60 ÷ 5 = 12 chocs

Tim 3 shares   3 x 12 = 36 chocs
Tom 2 shares   2 x 12 = 24 chocs

DIRECT PROPORTION:

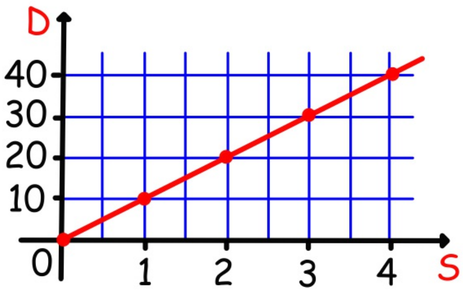
DISTANCE and SPEED are directly proportional

2 x speed results in 2 x distance travelled

One quantity is a multiple of the other.

GRAPH: a straight line through the origin

speed (S)	0	1	2	3	4
distance (D)	0	10	20	30	40



$D = 10 \times S$

Ten books cost £36.

(a) Find the cost of seven books.

(b) How many books for £54 ?

Rate: £3.60 per book

(a) 10 books ⟶ £36

1 book ⟶ £36 ÷ 10

7 books ⟶ £36 ÷ 10 x 7 = £25.20

(b) £36 ⟶ 10 books

£1 ⟶ 10 ÷ 36 books

£54 ⟶ 10 ÷ 36 x 54 = 15 books

or £54 ÷ £3.60

INVERSE PROPORTION:

TIME and SPEED are inversely proportional

2 x speed results in $^1/_2$ x time taken

The quantities multiply to the same product.

speed (S)	1	2	3	4
time (T)	12	6	4	3

$S \times T = 12$

A school has money to buy 50 books at £18 each.
Price increases to £20. How many books can be bought?

50 books at £18 each total money = 50 x £18 = £900

£900 at £20 each Nº books = 900 ÷ 20 = 45 books

OR

18 £/book ⟶ 50 books

1 £/book ⟶ 50 x 18 (900 books)

20 £/book ⟶ 50 x 18 ÷ 20 = 45 books