MATHEMATICS

S1+S2 COURSE NOTES

LEVEL 3/4

I

MATHS

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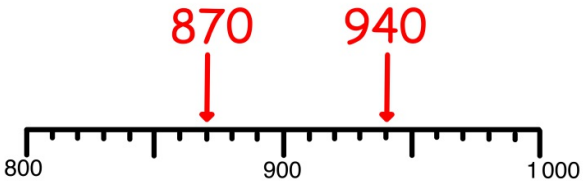
CHAPTER 1: NUMBER

In Words: place value of figures required.

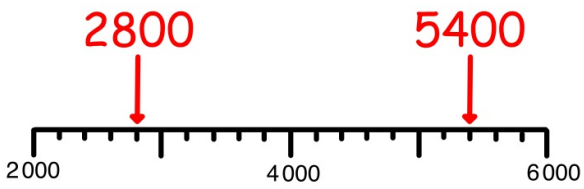
Th	H	T	U
4	5	7	6	9

forty five thousand, seven hundred and sixty nine.

Reading Scales:



each division 10



each division 200

CALCULATIONS:

Th	H	T	U
	8	3	7
	9	6	2
+	5	4	9
2	3	4	8
2	1	1	

Th	H	T	U
	$^{8}\not{9}$	$^{1}4$	$^{7}\not{8}^{1}3$
-	5	7	6	9
	3	7	1	4

$$5\overline{)6^{1}7^{2}5} = 135$$

$$534 \times 6 = 3204$$

3 2 2

POWERS

index → n

base → $a^n = a \times a \times \ldots\ldots \times a$ to n terms

" 2 to the power of 5 "

(i) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

(ii) $2^7 = 2^5 \times 2 \times 2 = 32 \times 2 \times 2 = 128$

(i) 2^3
$= 2 \times 2 \times 2$
$= 8$

(ii) $(7 - 3)^2$
$= 4^2$
$= 4 \times 4$
$= 16$

(iii) $7^2 - 3^2$
$= 7 \times 7 - 3 \times 3$
$= 49 - 9$
$= 40$

EVALUATE EXPRESSIONS

Replace the letter by its value.

$a = 2$, $b = 3$, $c = 5$

(1) $b^2 - a^3$
$= b \times b - a \times a \times a$
$= 3 \times 3 - 2 \times 2 \times 2$
$= 9 - 8$
$= 1$

(2) ac^2
$= a \times c \times c$
$= 2 \times 5 \times 5$
$= 2 \times 25$
$= 50$

(3) $(ac)^2$
$= (a \times c)^2$
$= (2 \times 5)^2$
$= 10^2$
$= 100$

SQUARE ROOT

Square and square root are inverse processes.

$3^2 = 9$

$\sqrt{9} = 3$

ROOTS 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 ....

SQUARES 1 , 4 , 9 , 16 , 25 , 36 , 49 , 64 .....

approximation: $\sqrt{40}$ lies between 6 and 7.

Solve Equations:

(i) $x^2 = 36$

$x = \sqrt{36}$

$x = 6$

(ii) $\sqrt{x} = 6$

$x = 6^2$

$x = 36$

SUM (add) 12 + 4 = 16

DIFFERENCE (subtract) 12 - 4 = 8

PRODUCT (multiply) 12 x 4 = 48

QUOTIENT (divide) 12 ÷ 4 = 3

MULTIPLE: a product by multiplying by an integer.

multiples of 6: 6 , 12 , 18 , 24 ...

multiples of 8: 8 , 16 , 24 , 36 ...

common multiples: 24 , 48 ...

lowest common multiple: 24 LCM = 24

FACTOR: pairs multiply to give the number.

12: $1 \times 12 = 2 \times 6 = 3 \times 4$

factors of 12: 1 , 2 , 3 , 4 , 6 , 12
factors of 18: 1 , 2 , 3 , 6 , 9 , 18
common factors: 1 , 2 , 3 , 6
highest common factor: 6 HCF = 6

PRIME NUMBER: has exactly 2 factors, itself and 1.

2, 3, 5, 7, 11, 13, 17... Note: 2 is the only even prime

PRODUCT OF PRIME FACTORS:

factors of 12: 1 , 2 , 3 , 4 , 6 , 12
prime factors of 12: 2 , 3
product of prime factors: $12 = 2 \times 2 \times 3$

divide successively by primes to reduce the number to 1.

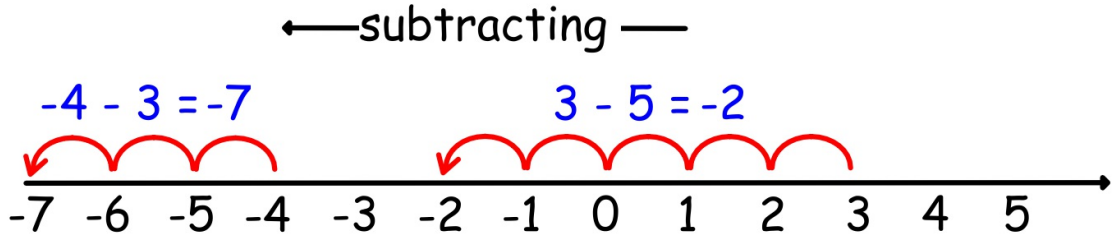
2	240
2	120
2	60
2	30
3	15
5	5
	1

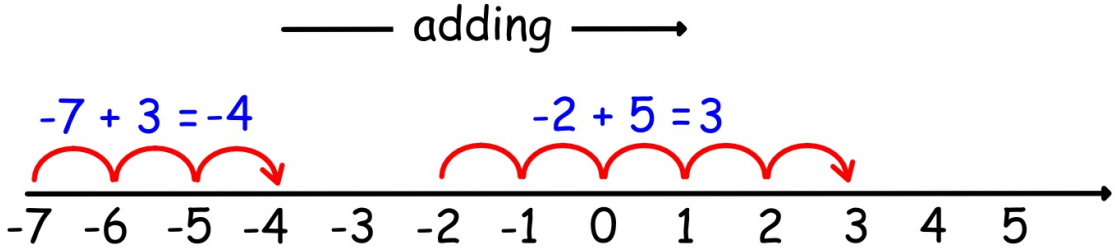
$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$= 2^4 \times 3 \times 5$ INDEX FORM

DIRECTED NUMBERS

Add and Subtract





$a + (-b) = a - b$

$a - (-b) = a + b$

(i) $7 + (-3) = 7 - 3 = 4$

(ii) $-5 + (-1) = -5 - 1 = -6$

(iii) $6 - (-2) = 6 + 2 = 8$

(iv) $-3 - (-5) = -3 + 5 = 2$

Multiply and Divide

P	x or ÷	P	=	P
N	x or ÷	N	=	P
P	x or ÷	N	=	N
N	x or ÷	P	=	N

$-3 \times (-6) = 18$ $\quad -18 \div (-6) = 3$

$3 \times (-6) = -18$ $\quad 18 \div (-6) = -3$

$-3 \times 6 = -18$ $\quad -18 \div 6 = -3$

ORDER OF CALCULATION

x and ÷ before + or -
change the order with brackets:
BRACKETS FIRST !

(i) $-6 + 4 \times (-3)$
$= -6 + (-12)$
$= -18$

(ii) $(-6 + 4) \times (-3)$
$= -2 \times (-3)$
$= 6$

EVALUATE EXPRESSIONS

Replace the letter by its value.

$a = -2$, $b = 3$, $c = -5$

(1) $5ab$
$= 5 \times a \times b$
$= 5 \times (-2) \times 3$
$= -30$

(2) $b^2 + ac$
$= b \times b + a \times c$
$= 3 \times 3 + (-2) \times (-5)$
$= 9 + 10$
$= 19$

(3) $\dfrac{c - b}{c + b}$
$= \dfrac{-5 - 3}{-5 + 3}$
$= \dfrac{-8}{-2}$
$= 4$

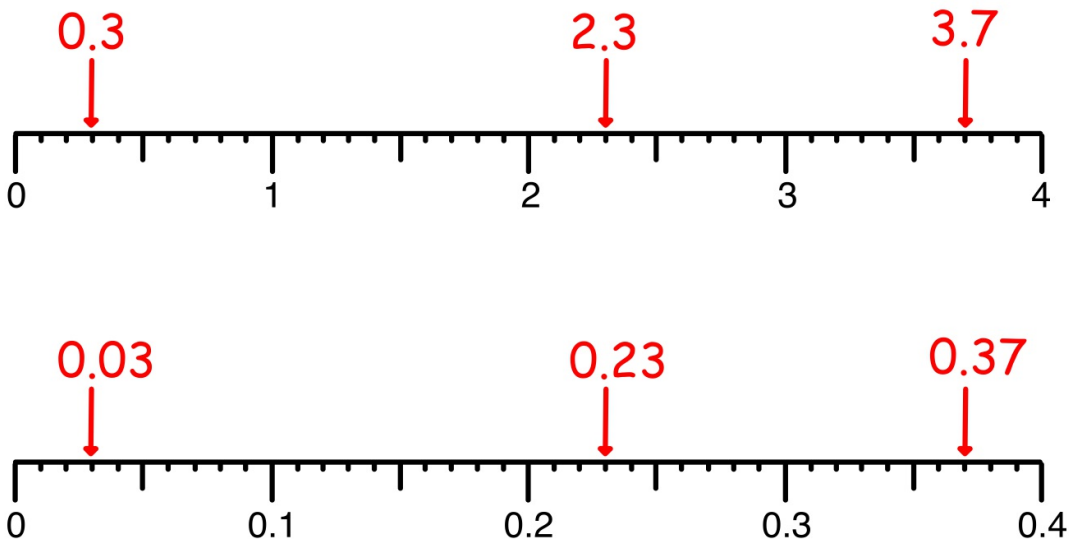
(4) $a(b + c)$
$= a \times (b + c)$
$= -2 \times (3 + (-5))$
$= -2 \times (-2)$
$= 4$

(5) $2c^2$
$= 2 \times c \times c$
$= 2 \times (-5) \times (-5)$
$= 2 \times 25$
$= 50$

(6) $(2c)^2$
$= (2 \times c)^2$
$= (2 \times (-5))^2$
$= (-10)^2$
$= 100$

CHAPTER 2: DECIMALS

READING SCALES



DECIMAL PLACES

the number of figures after the decimal point.

ROUNDING

(i) 12.649 (3DP) → 12.7 / 12.6 → 12.65

= 12.6 (1DP)

(ii) 3.6851 (4DP) → 3.69 / 3.68 → 3.685

= 3.69 (2DP)

SIGNIFICANT FIGURES

Indicate the accuracy of a measurement.

eg. 3400 cm = 34 m = 0.034 km

Same measurement, same accuracy: each 2 significant figures

Count the number of figures used but not zeros:
at the end of a whole number,
at the start of a decimal.

These zeros simply keep the place value of the digits.

Examples:

0.030	2 sig fig	30100	3 sig fig
5.030	4 sig fig	30100.0	6 sig fig

ROUNDING:

Round to the place value of the number of significant figures required

Examples: rounding to 2 significant figures.

(i) TH H T U t
5751.4 = 5800 nearest HUNDRED

(ii) U t h th
0.057514 = 0.058 nearest THOUSANDTH

NOTE: 0.058000 is wrong

ADD and SUBTRACT

remember place value

(i) 8.37 + 9.6 + 5.49

T	U	t	h
	8	3	7
	9	6	0
+	5	4	9
2	3	4	6
2	1	1	

(ii) 9.48 - 5.764

	U	t	h	th
	$^{8}\not{9}$	$^{1}4$	$^{7}\not{8}$	$^{1}0$
-	5	7	6	4
	3	7	1	6

MULTIPLY and DIVIDE

(i) 4.547 x 6

	4	5	4	7
				x 6
2	7	2	8	2
	3	2	4	

(ii) 27.282 ÷ 6

		0	4	5	4 7
6	$\vert$	$^{2}2$	$^{3}7$	$^{2}2$	$^{4}8$ $^{2}2$

multiply by 10s

(i) $3.42 \times 10 = 34.2$

(ii) $5.87 \times 100 = 587$

(iii) $0.062 \times 100 = 6.2$

divide by 10s

(i) $34.2 \div 10 = 3.42$

(ii) $587. \div 100 = 5.87$

(iii) $006.2 \div 100 = 0.062$

(1) $3.12 \div 0.003$

$= 3120 \div 3$

$= 1040$

remove decimal point from divisor: multiply both numbers by 1000

(2) $3.12 \div 3000$

$= 1.04 \div 1000$

$= 0.00104$

divide by 3 then by 1000

(3) 3.12×3000

$= 3120 \times 3$

$= 9360$

multiply by 1000 then by 3

CALCULATOR:

Read and decide the operation(s) required + , - , x , ÷

Write the calculation(s).

(1) Carpet costs £24.52 per m^2. Calculate the cost of 18.5 m^2.

£24.52 x 18.5 = £453.62

(2) 4.8 litres of paint covers 58.8 m^2.

How many m^2 per litre ?

58.8 ÷ 4.8 = 12.25 m^2

(3) On a diet John drops from 105.4 kg to 96.8 kg.

How much weight did he loose ?

105.4 - 96.8 = 8.6 kg

FOREIGN EXCHANGE

EXCHANGE RATE: amount of foreign currency for £1.

£1 = $1.48

Change £30 to $

$30 x 1.48 = $44.40

Change $250 to £

£250 ÷ 1.48

= £168.91891...

= £168.92

STANDARD FORM (SCIENTIFIC NOTATION)

Used to write very large and very small numbers.

Form $a \times 10^n$

$1 \leq a < 10$ ie. between 1 and 10, excluding 10

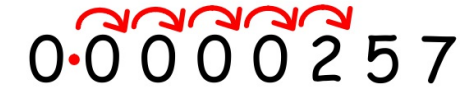
n is an INTEGER ie. ...-3,-2,-1,0,1,2,3...

Place the decimal point after the first non-zero digit.
Count the number of places the decimal point moves.

$257000 = 2.57 \times 10^5$

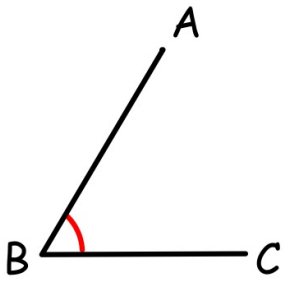
negative power - small number between 0 and 1,
the point moves to the right.

$0.0000257 = 2.57 \times 10^{-5}$



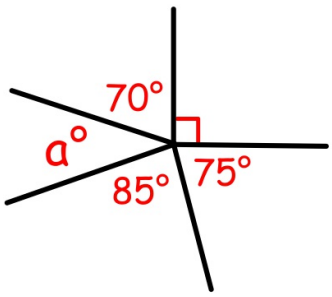
CHAPTER 3: ANGLES

NAMING ANGLES: 3 capital letters and an angle sign
the middle letter is the VERTEX.



arms: AB and BC
vertex: B
angle: ∠ABC

COMPLETE TURN:
360° around a point

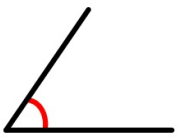


70
85
75
+ 90
320

360
- 320
40

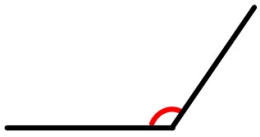
a = 40

DESCRIPTIONS (TYPES):



acute angle:
between 0° and 90°

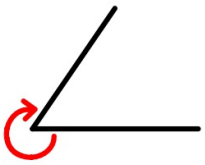
right angle:
90°

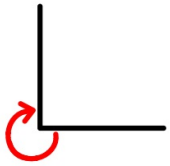


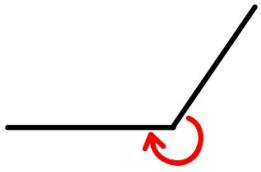
obtuse angle:
between 90° and 180°

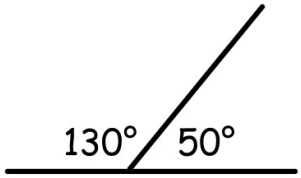
straight angle:
180°

Reflex angle: between 180° and 360°

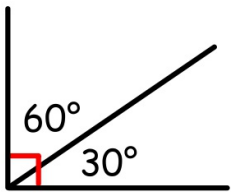




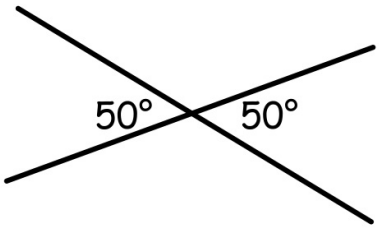




supplementary

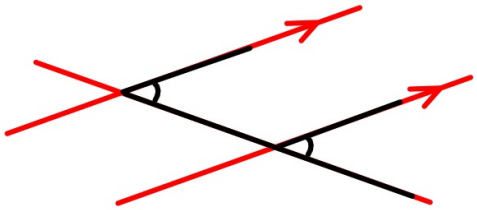


complementary

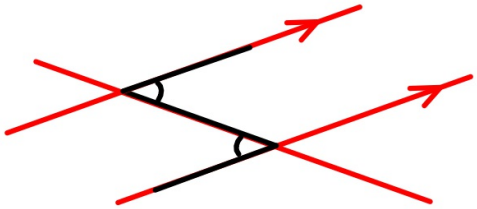


vertically opposite
(X shape)

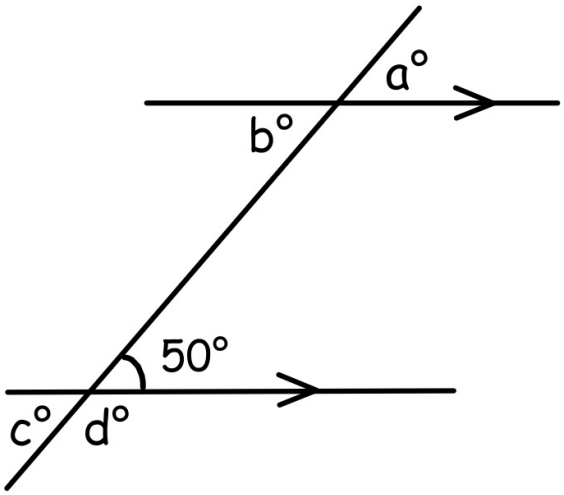
PARALLEL LINES:



corresponding angles equal
(F shape)



alternate angles equal
(Z shape)



a = 50 corresponding (F shape)

b = 50 alternate (Z shape)

c = 50 vertically opposite
(X shape)

d = 130 supplementary

$$\begin{array}{r} 180 \\ -\ 50 \\ \hline 130 \end{array}$$

CHAPTER 4: ALGEBRA

SHORT FORMS

multiplication is repeated addition

(i) $7 + 7 + 7 + 7 + 7 + 7 = 6 \times 7$

(ii) $a + a + a + a + a + a = 6 \times a = 6a$

(iii) $a^2 + a^2 + a^2 + a^2 + a^2 + a^2 = 6 \times a^2 = 6a^2$

a power is repeated multiplication

(i) $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6$

(ii) $a \times a \times a \times a \times a \times a = a^6$

"Like Terms" can be added/subtracted.
(same letter combinations)

(1) $5a - 2b + 3a$
$= 5a + 3a - 2b$
$= 8a - 2b$

(2) $5ab - 2ba + ab$
$= 5ab - 2ab + 1ab$
$= 4ab$

(3) $7a^2 - 4a - 6a^2$
$= 7a^2 - 6a^2 - 4a$
$= a^2 - 4a$

Multiplication combines letters and numbers.

(4) $2a^2 \times 4ab$
$= 2 \times a \times a \times 4 \times a \times b$
$= 2 \times 4 \times a \times a \times a \times b$
$= 8a^3b$

(5) $3c^2 \times 2c - 5 \times c \times c \times d$
$= 6c^3 - 5c^2d$

BRACKET BREAKING

$$a \times (b + c) = a \times b + a \times c$$

multiply each term in the brackets

(1) $6(2w - 3y)$

$= 12w - 18y$

(2) $8p(2p + r)$

$= 16p^2 + 8pr$

remove brackets then simplify:

(3) $2(3p + 5) - 7$

$= 6p + 10 - 7$

$= 6p + 3$

(4) $n(4n - 3) + n^2$

$= 4n^2 - 3n + n^2$

$= 5n^2 - 3n$

FACTORISATION

$$a \times b + a \times c = a \times (b + c)$$

To FULLY factorise a is the Highest Common Factor

(1) $12w - 18y$

$= 6(2w - 3y)$

(2) $4n^2 + 5mn - n$

$= n(4n + 5m - 1)$

(3) $16p^2 + 8pr$

$= 8p(2p + r)$

(4) $1.4 \times 3.6 - 1.4 \times 1.6$

$= 1.4 \times (3.6 - 1.6)$

$= 1.4 \times 2$

$= 2.8$

CHAPTER 5: INFORMATION HANDLING 1

AVERAGES:

$$\text{mean} = \frac{\text{total of all results}}{\text{number of results}}$$

median: the middle result of the ordered results
(if an even number of results - find the mean of the middle two results)

mode: the most common result

SPREAD:

range = highest result - lowest result

19,14,15,19,16,22,16,20,24,14,25,18,27,12,16

mean = $^{277}/_{15}$ = 18.466... = 18.5

15 ordered results: 8th result in the middle

12,14,14,15,16,16,16,18,19,19,20,22,24,25,27

median = 18

mode = 16

range = 27 - 12 = 15

median for even number of results:

16 ordered results: 8th/9th results in the middle

12,14,14,15,16,16,16,18,19,19,20,22,24,25,27,30

median = (18+19) ÷ 2 = 18.5

MEAN PROBLEMS Total = mean x number of results

(1) 15 girls, mean height 1.2m
10 boys, mean height 1.4m

Find the mean height of the group of 25 children ?

1.2 m x 15 = 18 m
1.4 m x 10 = 14 m
total height 32 m

mean = 32 ÷ 25 = 1.28 m

(2) 4 boys have mean weight 60 kg
When John is included the mean weight is 62 kg

Find the weight of John.

60 Kg x 4 = 240 Kg
62 Kg x 5 = 310 Kg
310 - 240 = 70 Kg

FREQUENCY TABLES

If there is a lot of data the information is easier to present in a frequency distribution table.

TEST SCORES: 1,1,1,2,2,2,2,2,3,3,3,3,3,3,3,4,4,4,5,5

result	tally	frequency
1	𝍸	3
2	𝍸	5
3	𝍸 ||	7
4	|||	3
5	||	2

MEAN FROM A FREQUENCY TABLE:

A result x frequency column is included.

$$\text{mean} = \frac{\text{total of: result} \times \text{frequency}}{\text{total frequency}}$$

result	frequency	result x frequency
1	3	3
2	5	10
3	7	21
4	3	12
5	2	10
TOTALS	20	56

mean = 56 ÷ 20
= 2.8

GROUPED FREQUENCY TABLES

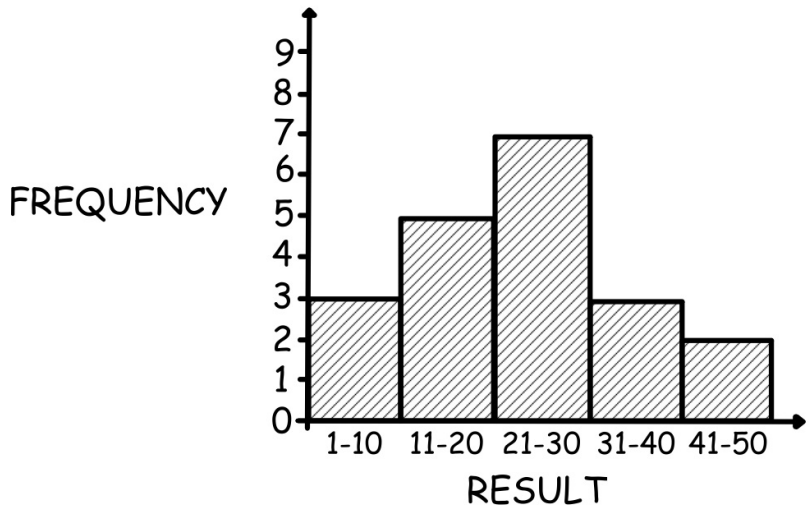
Data can be grouped into class intervals

TEST SCORES: 3,5,8 , 11,13,17,17,19,
22,23,24,25,26,28,29,
33,38,39 , 44,47

result	tally	frequency
1 - 10	III	3
11 - 20	𝍸	5
21 - 30	𝍸 II	7
31 - 40	III	3
41 - 50	II	2

The results can be shown on a frequency diagram.

The MODAL CLASS has the highest frequency.



MODAL CLASS is 21-30

CHAPTER 6: TIME and SPEED

CLOCK TIME:

12 hour	6.25 am	6.25 pm	noon	midnight
24 hour	06 25	18 25	12 00	00 00 24 00

Note: 6.25 am is 6hr 25min into the day

6·25 hr is 6hr 15min

($6^1/_4$ hr)

TIME DIFFERENCES

(1) 8.25 am to 4.10 pm

08 25 to 09 00		35 min
09 00 to 16 10	7 hr	10 min
	7 hr	45 min

(2) 10.35 pm to 2.20 am

22 35 to 24 00	1 hr	25 min
00 00 to 02 20	2 hr	20 min
	3 hr	45 min

SPEED

$S = \frac{D}{T}$

$D = ST$

$T = \frac{D}{S}$

minutes ÷ 60 = hours

hours x 60 = minutes

(1) Travel 240 miles in 4 hours 10 min.
Find the average speed.

$S = \frac{D}{T}$

= 240 ÷ 4.166666666

= 57.6 mph

10 ÷ 60 + 4

240 ÷ ANS =

(2) How far is travelled at 36 mph for 4 hours 35 min. ?

$D = ST$

= 36 x 4.583333333

= 165 miles

35 ÷ 60 + 4

(3) How long does it take to travel 100 miles at 24 mph ?

$T = \frac{D}{S}$

= 100 ÷ 24

= 4.166666666 hours

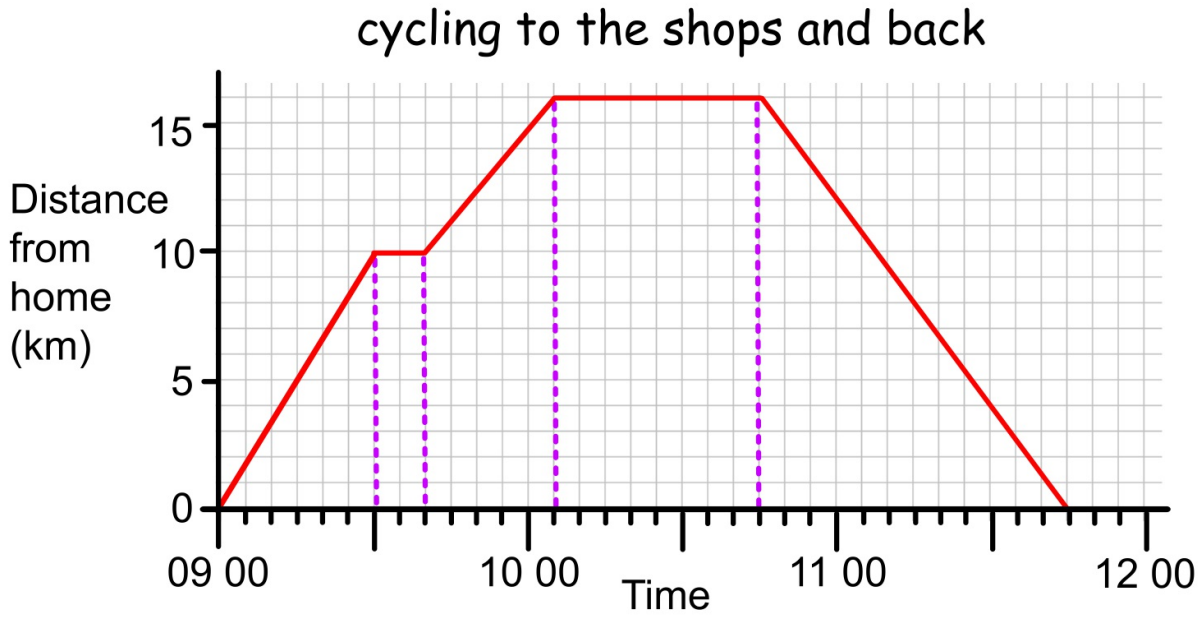
= 4 hours 10 min

- 4

0.1666... x 60 = 10

GRAPHS

For a distance/time graph, the slope is the speed.
The faster the speed the steeper the graph.



The graph tells a story of the journey:

How far is it to the shops ? 16 km

How long was he shopping ? 8 divisions 40 min

(1 division = 60 min ÷ 12 = 5 min)

At what time did he set off for home ? 10 45

How long did he stop to repair a puncture ?

2 divisions 10 min