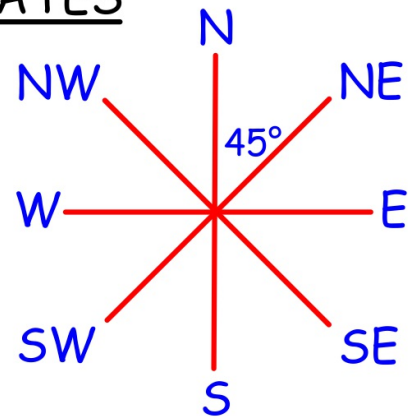
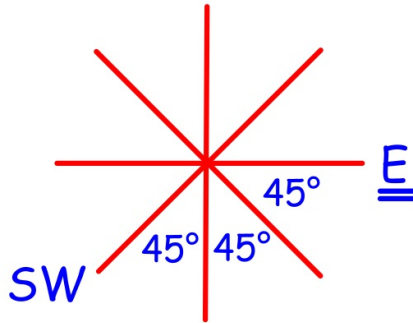
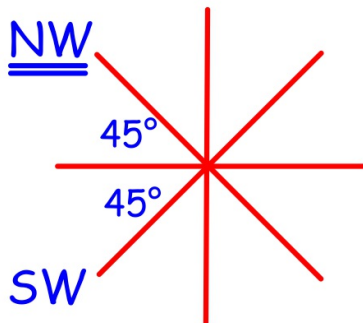


CHAPTER 7: SCALES and COORDINATES

8 POINT COMPASS

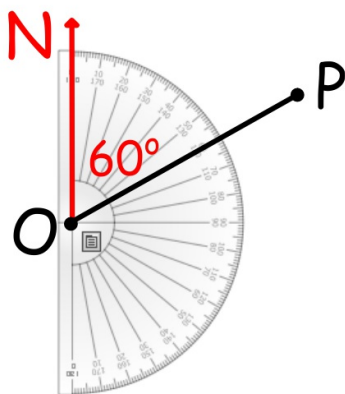


Facing SW, what direction do I face after turning:
(a) clockwise 90° (b) anticlockwise 135° ?

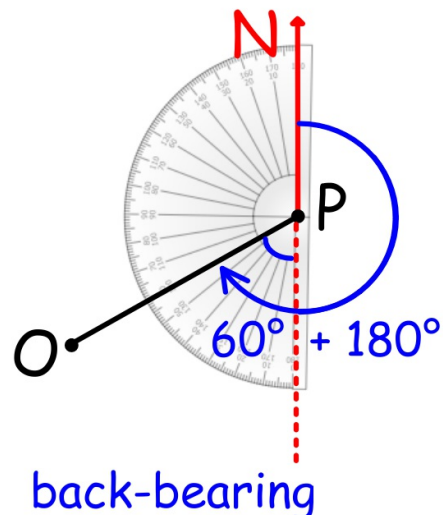


THREE FIGURE BEARINGS

The angle of turning clockwise from North.



P from O is 060°



back-bearing

O from P is 240°

The bearing and back-bearing differ by 180°

SCALE AS RATIO: REPRESENTATIVE FRACTION

$$1 \text{ cm} = 2 \text{ km}$$

$$\begin{aligned} & 1 \text{ cm} : 2 \text{ km} \\ &= 1 \text{ cm} : 2000 \text{ m} \\ &= 1 \text{ cm} : 200000 \text{ cm} \\ &= 1 : 200000 \end{aligned}$$

(i) $1 : 200000$

map distance 7.2 cm

real distance: ?

$$\begin{aligned} & 7.2 \text{ cm} \times 200000 \\ &= 1440000 \text{ cm} \\ &= 14400 \text{ m} \\ &= 14.4 \text{ km} \end{aligned}$$

(ii) $1 : 50000$

real distance 3.4 km

map distance: ?

$$\begin{aligned} & 3.4 \text{ km} \div 50000 \\ &= 3400 \text{ m} \div 50000 \\ &= 340000 \text{ cm} \div 50000 \\ &= 34 \text{ cm} \div 5 \\ &= 6.8 \text{ cm} \end{aligned}$$

(iii) real distance 7 m

map distance 3.5 cm

representative fraction: ?

$$\begin{aligned} & 3.5 \text{ cm} : 7 \text{ m} \\ &= 3.5 \text{ cm} : 700 \text{ cm} \\ &= 1 : 200 \end{aligned}$$

by $700 \div 3.5$

SCALE DRAWING

Suitable scale for a big diagram.

Neat and accurate, labelled with lengths and angles

Write the measurements made and all the calculations

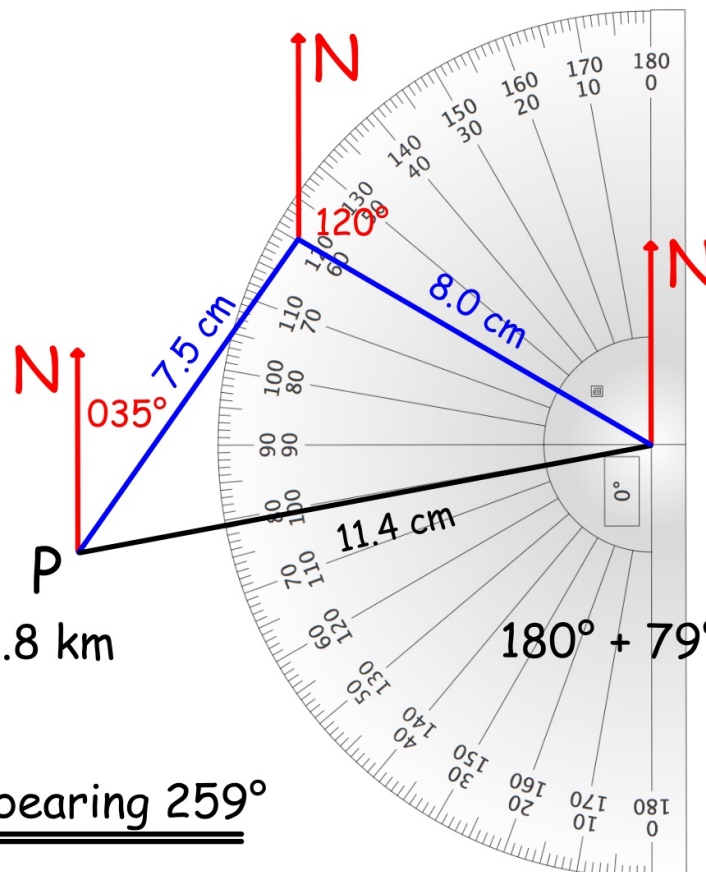
A ship leaves port and sails for 15 km on bearing 035° and then for 16 km on bearing 120°.

Find the distance and bearing to return to port.

$$1 \text{ cm} = 2 \text{ km}$$

$$15.0 \div 2 = 7.5 \text{ cm}$$

$$16.0 \div 2 = 8.0 \text{ cm}$$



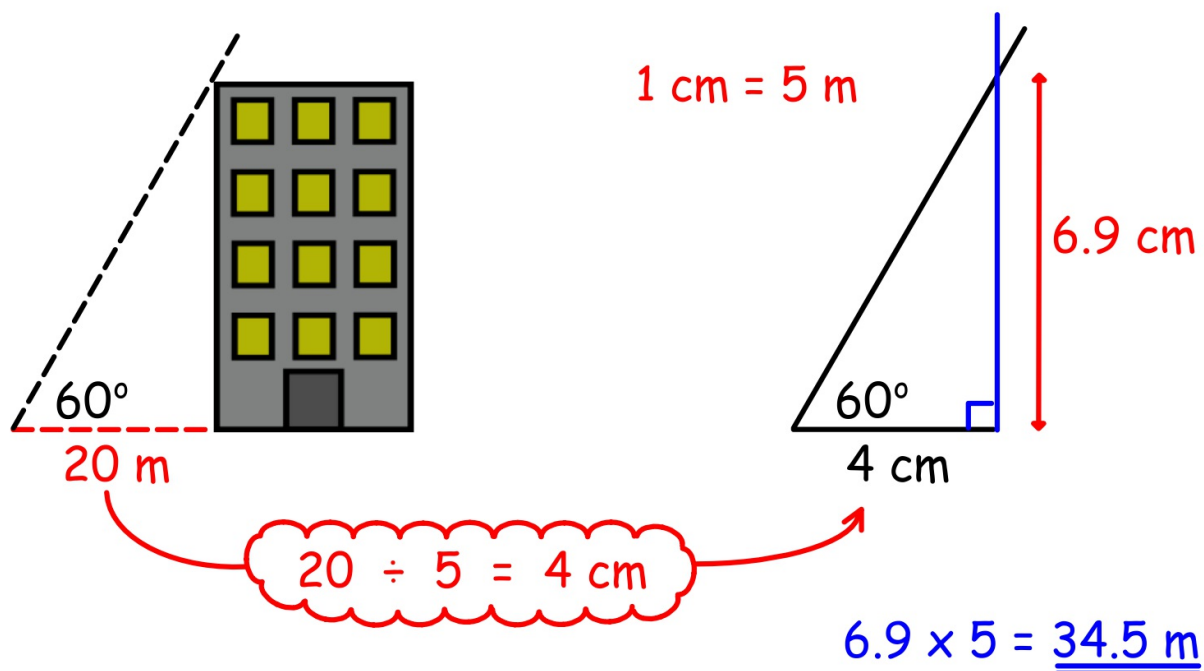
$$11.4 \times 2 = 22.8 \text{ km}$$

$$180^\circ + 79^\circ = 259^\circ$$

22.8 km on bearing 259°

SCALE DRAWING

Find the height of the building.



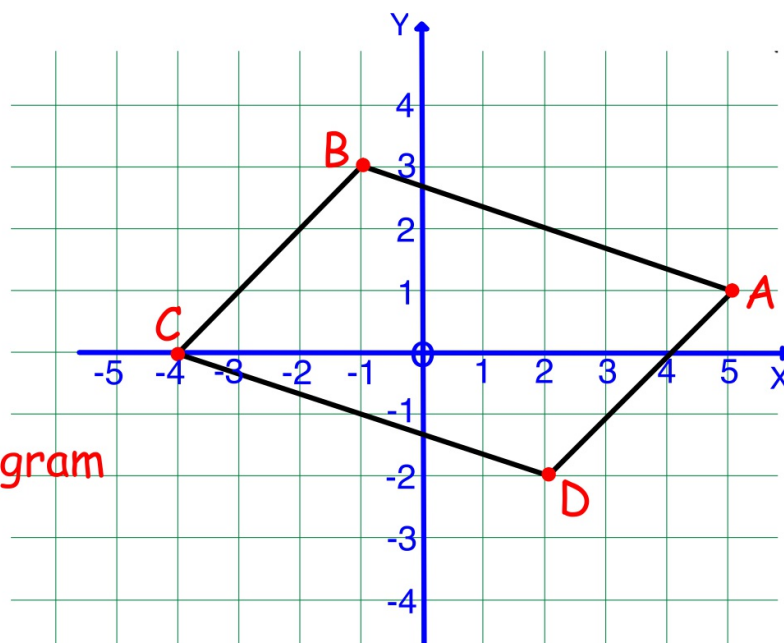
COORDINATES

A (5,1) x-coordinate is 5
y-coordinate is 1

B (-1,3)

C (-4,0)

D (2,-2)



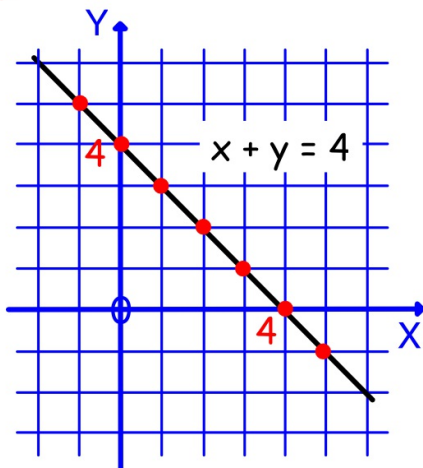
ABCD is a parallelogram

LINES

The equation gives a rule connecting the x and y coordinates of any point on the line.

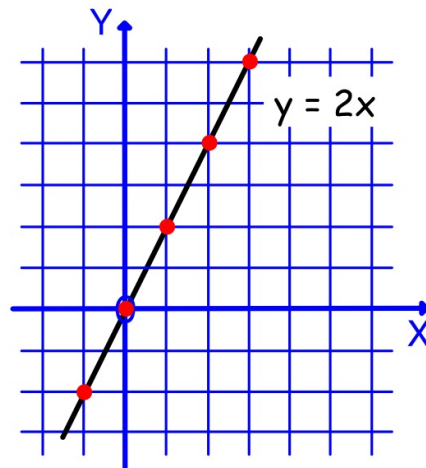
$$x + y = 4$$

x	-1	0	1	2	3	4	5
y	5	4	3	2	1	0	-1



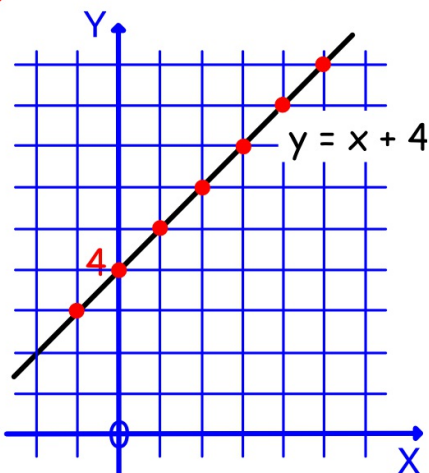
$$y = 2x$$

x	-1	0	1	2	3
y	-2	0	2	4	6

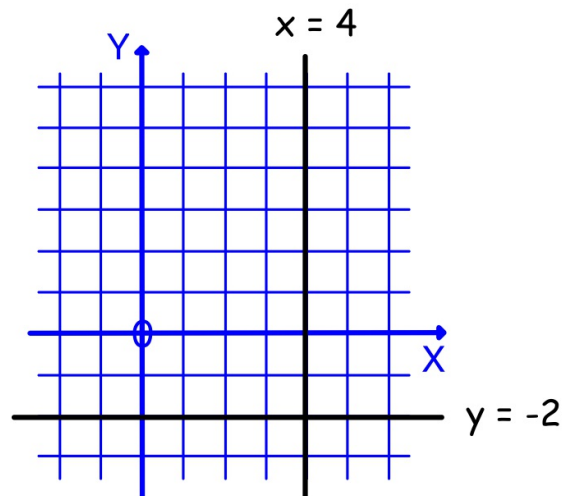


$$y = x + 4$$

x	-1	0	1	2	3	4	5
y	3	4	5	6	7	8	9



VERTICAL and HORIZONTAL lines



CHAPTER 8: EQUATIONS and INEQUATIONS

SIMPLE EQUATIONS: 'cover-up' type

isolate the term with the letter

$$(1) \quad 2x - 3 = 11$$

$$2x = 14$$

$$x = 7$$

$$(2) \quad 21 = 6 + 5a$$

$$5a = 15$$

$$a = 3$$

$$(3) \quad 17 - 3w = 5$$

$$3w = 12$$

$$w = 4$$

$$(4) \quad \frac{1}{2}h - 3 = 2$$

$$\frac{1}{2}h = 5$$

$$h = 10$$

LETTERS ON BOTH SIDES - get letters on one side

$$(1) \quad 5x - 2 = 2x + 10$$

$$3x - 2 = 10$$

$$3x = 12$$

$$x = 4$$

subtract $2x$ from each side

add 2 to each side

divide each side by 3

$$(2) \quad 2 + 3n = 5 - 4n$$

$$2 + 7n = 5$$

$$7n = 3$$

$$n = \frac{3}{7}$$

add $4n$ to each side

subtract 2 from each side

divide each side by 7

EQUATIONS and negatives

$$\begin{array}{ll} (1) \quad 17 - 2y = 3 & \text{subtract 17 from each side} \\ \quad \quad -2y = -14 & \text{divide each side by -2} \\ \quad \quad \quad y = 7 & \end{array}$$

$$\begin{array}{ll} (2) \quad 8 + 2n = 6 - 3n & \text{add 3n to each side} \\ \quad \quad 8 + 5n = 6 & \text{subtract 8 from each side} \\ \quad \quad \quad 5n = -2 & \text{divide each side by 5} \\ \quad \quad \quad \quad n = -\frac{2}{5} & \end{array}$$

EQUATIONS with fractions

first remove fractions: multiply by the denominator.

$$\begin{array}{ll} (1) \quad \frac{x}{2} = 5 & (2) \quad \frac{x-3}{4} = -2 \\ \quad \quad \quad \text{multiply by 2} & \quad \quad \quad \text{multiply by 4} \\ \quad \quad \quad x = 10 & \quad \quad \quad x - 3 = -8 \\ & \quad \quad \quad x = -5 \end{array}$$

NOTE: same equations

$$\frac{1}{2}x = 5$$

$$\frac{1}{4}(x-3) = -2$$

BRACKET BREAKING

$$a \times (b + c) = a \times b + a \times c$$

$$(1) \quad 3p(2p + r) \\ = 6p^2 + 3pr$$

$$(2) \quad 2a(3a - b + 5) \\ = 6a^2 - 2ab + 10a$$

signs change when multiplying by a negative term:

$$(3) \quad -3(2w - 3y) \\ = -6w + 9y$$

$$(4) \quad -n(4n + 5m) \\ = -4n^2 - 5mn$$

EXPRESSIONS: remove brackets then simplify.

$$(1) \quad 2a + 3a(2 - 3a) \\ = 2a + 6a - 9a^2 \\ = 8a - 9a^2$$

no sign change

$$(2) \quad 5 - 3(2a - 3) \\ = 5 - 6a + 9 \\ = 14 - 6a$$

sign change

EQUATIONS: remove brackets then solve.

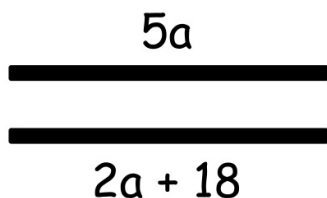
$$(1) \quad 2(w + 6) = 5(w - 3) \\ 2w + 12 = 5w - 15 \\ 12 = 3w - 15 \\ 27 = 3w \\ w = 9$$

$$(2) \quad 3y = 14 - 2(y - 3) \\ 3y = 14 - 2y + 6 \\ 3y = 20 - 2y \\ 5y = 20 \\ y = 4$$

PROBLEM SOLVING

Form an equation and solve.

- (1) The two rods are the same length.
Find the length of the rods.

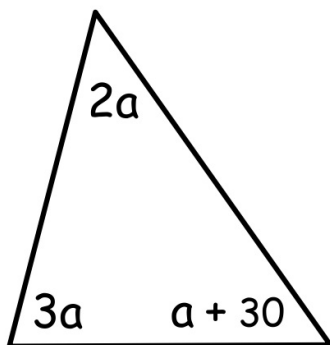


$$\begin{aligned}5a &= 2a + 18 \\3a &= 18 \\a &= 6\end{aligned}$$

$$\begin{aligned}5a &= 5 \times 6 = 30 \\2a + 18 &= 2 \times 6 + 18 = 30\end{aligned}$$

30 cm

- (2) Find the size of the angles.



$$\begin{aligned}3a + 2a + a + 30 &= 180 \\6a + 30 &= 180 \\6a &= 150 \\a &= 25\end{aligned}$$

$$\begin{aligned}3a &= 3 \times 25 = 75 \\2a &= 2 \times 25 = 50 \\a + 30 &= 25 + 30 = \underline{55} \\ \text{check: } &180\end{aligned}$$

50° , 55° , 75°

INEQUALITIES (INEQUATIONS)

$>$ greater than eg. $7 > 3$

\geq greater than or equal to

$<$ less than eg. $3 < 7$

\leq less than or equal to

solve for $x = 1, 2, 3, 4, 5, 6$

$$(1) \quad x > 4 \\ x = 5, 6$$

$$(2) \quad x \geq 4 \\ x = 4, 5, 6$$

$$(3) \quad x < 4 \\ x = 1, 2, 3$$

follow the same rules as equations

$$(1) \quad 5x - 4 < 6 \\ 5x < 10 \\ x < 2$$

$$(2) \quad 2x + 7 \leq 1 \\ 2x \leq -6 \\ x \leq -3$$

$$(3) \quad 5x < 3x + 10 \\ 2x < 10 \\ x < 5$$

$$(4) \quad 2x \geq 20 - 3x \\ 5x \geq 20 \\ x \geq 4$$

\times or \div by a negative: reverse the inequality sign

$$(i) \quad 5x \geq 30 \\ x \geq 6$$

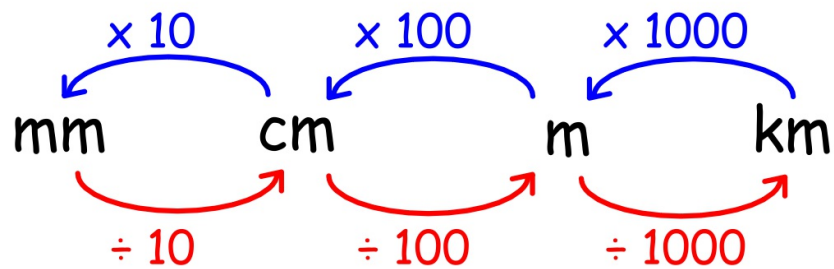
no sign change

$$(ii) \quad -5x \geq 30 \\ x \leq -6$$

sign change

CHAPTER 9: LENGTH and PYTHAGORAS' THEOREM

LENGTH UNITS multiply going to smaller units



divide going to larger units

$$18 \text{ mm} \div 10 = 1.8 \text{ cm}$$

$$0.7 \text{ m} \times 100 = 70 \text{ cm}$$

$$3.4 \text{ m} \times 100 = 340 \text{ cm}$$

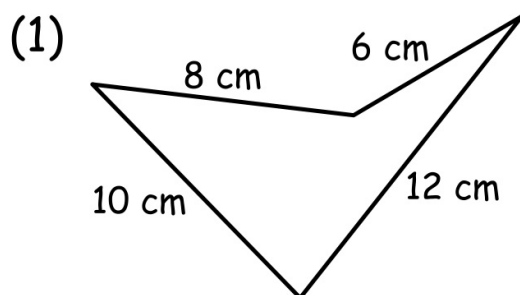
$$520 \text{ cm} \div 100 = 5.20 \text{ m}$$

$$40 \text{ m} \div 1000 = 0.040 \text{ km}$$

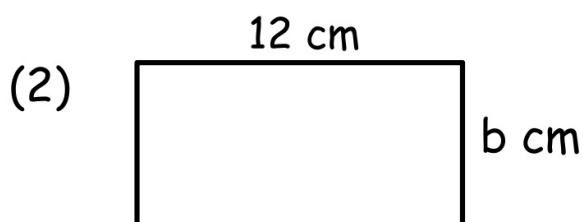
$$0.2 \text{ km} \times 1000 = 200 \text{ m}$$

PERIMETER

The total distance around the outside edge.



$$\begin{aligned} P &= 12 + 10 + 8 + 6 \\ &= 36 \text{ cm} \end{aligned}$$

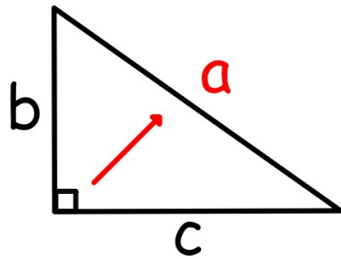


$$\begin{aligned} P &= 12 + 12 + b + b \\ 40 &= 24 + 2b \\ 2b &= 16 \\ b &= 8 \end{aligned}$$

rectangle perimeter 40 cm

PYTHAGORAS' THEOREM

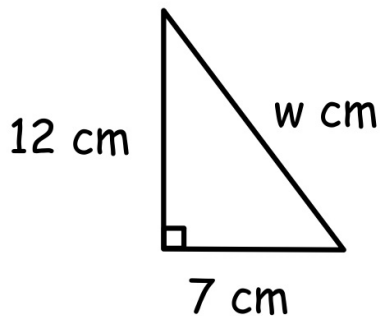
For right-angled triangles only:



$$a^2 = b^2 + c^2$$

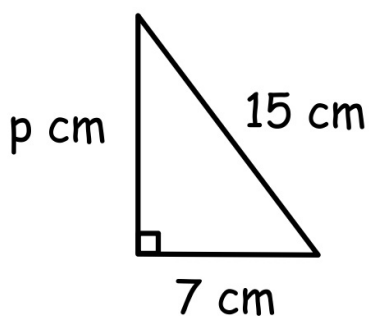
HYPOTENUSE (largest side) is opposite the 90° angle.

BIGGEST SIDE



$$\begin{aligned}w^2 &= 12^2 + 7^2 \\&= 144 + 49 \\&= 193 \\w &= \sqrt{193} \\&= 13.892... \\w &\approx 13.9\end{aligned}$$

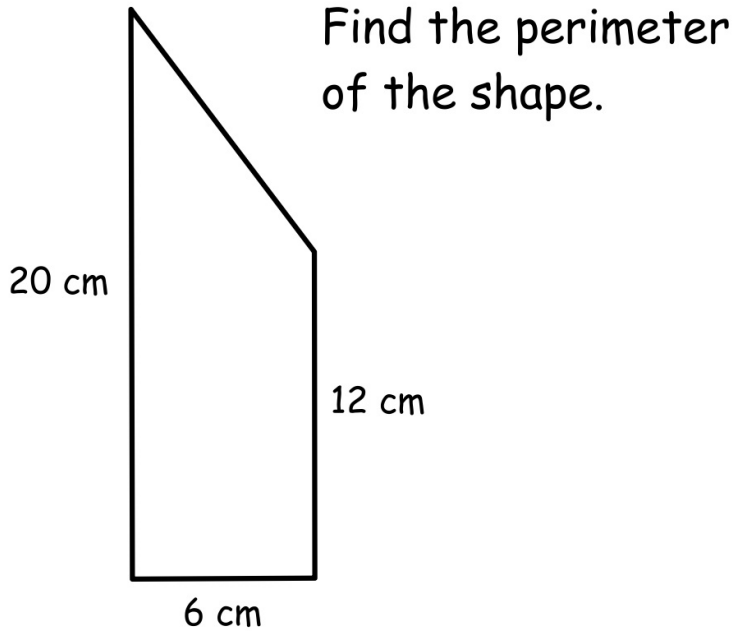
SMALLER SIDE



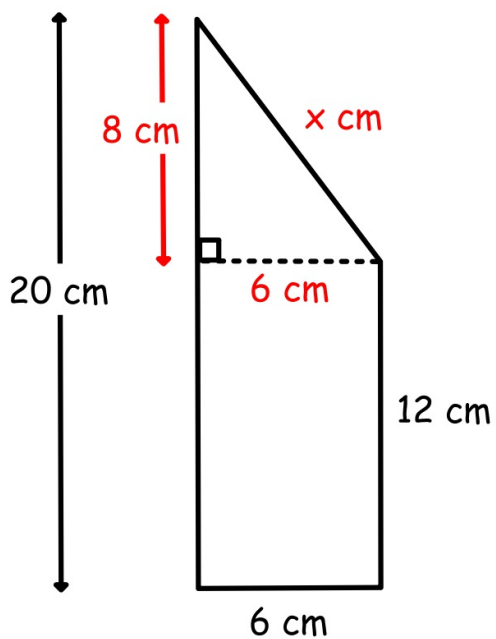
$$\begin{aligned}p^2 &= 15^2 - 7^2 \\&= 225 - 49 \\&= 176 \\p &= \sqrt{176} \\&= 13.266... \\p &\approx 13.3\end{aligned}$$

APPLYING PYTH. THM.

Identify the right angled triangle.



height of triangle $20\text{ cm} - 12\text{ cm} = 8\text{ cm}$



$$\begin{aligned}x^2 &= 8^2 + 6^2 \\&= 64 + 36 \\&= 100 \\x &= 10\end{aligned}$$

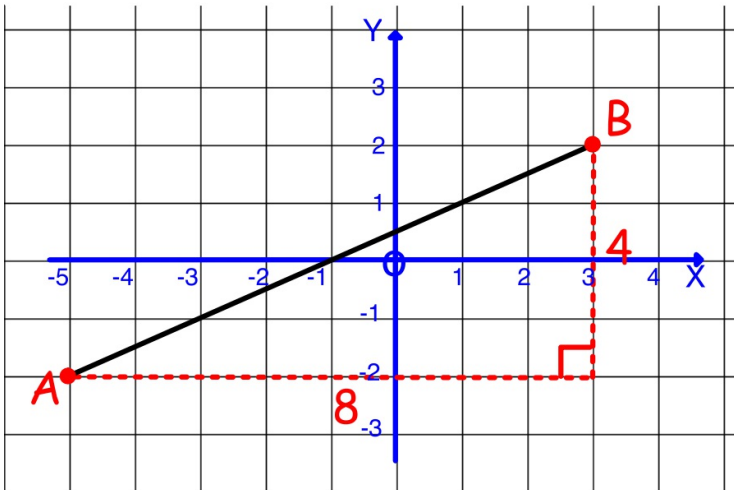
$$\begin{aligned}P &= 20\text{cm} + 6\text{cm} + 12\text{cm} + 10\text{cm} \\&= \underline{\underline{48\text{ cm}}}\end{aligned}$$

DISTANCE BETWEEN TWO POINTS

Plot the points.

Construct the right-angled triangle around them.

A (-5,-2) and B (3,2)



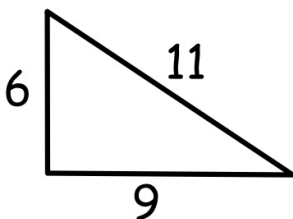
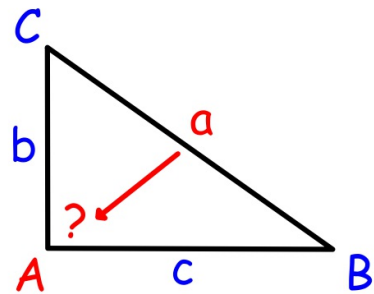
$$\begin{aligned} AB^2 &= 8^2 + 4^2 \\ &= 64 + 16 \\ &= 80 \end{aligned}$$

$$\begin{aligned} AB &= \sqrt{80} \\ &= 8.944\dots \end{aligned}$$

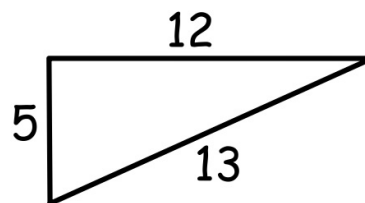
$$AB \approx 8.9 \text{ units}$$

CONVERSE OF PYTH. THM.

if $a^2 = b^2 + c^2$
then $\triangle ABC$ is right-angled



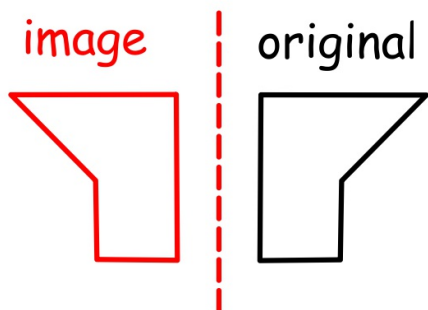
$6^2 + 9^2 \neq 11^2$
 \triangle is not right-angled



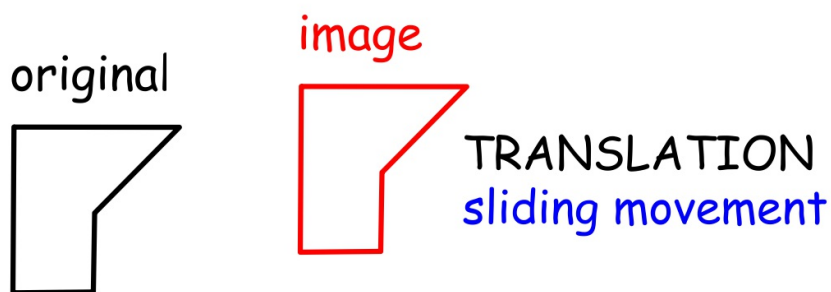
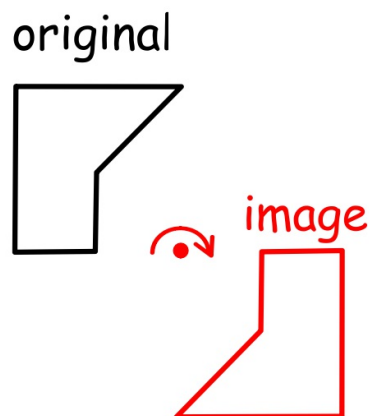
$5^2 + 12^2 = 13^2$
 \triangle is right-angled

CHAPTER 10: TRANSFORMATIONS

REFLECTION
folding over a line



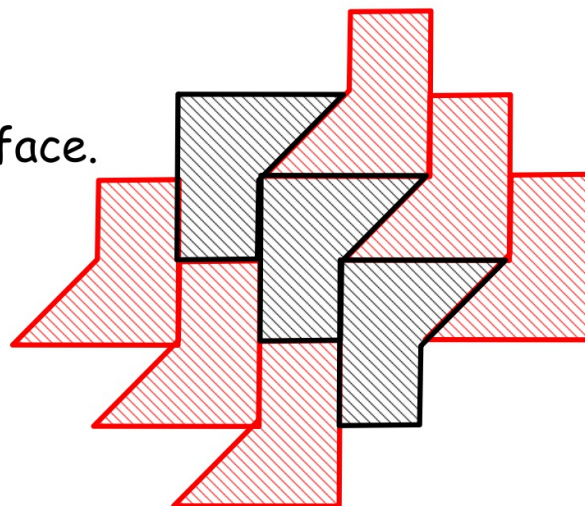
ROTATION
turning about a point



TILINGS

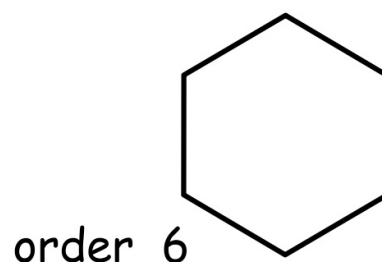
CONGRUENT tiles cover a surface.
(identical)

No gaps or overlap.
Can extend in any direction.



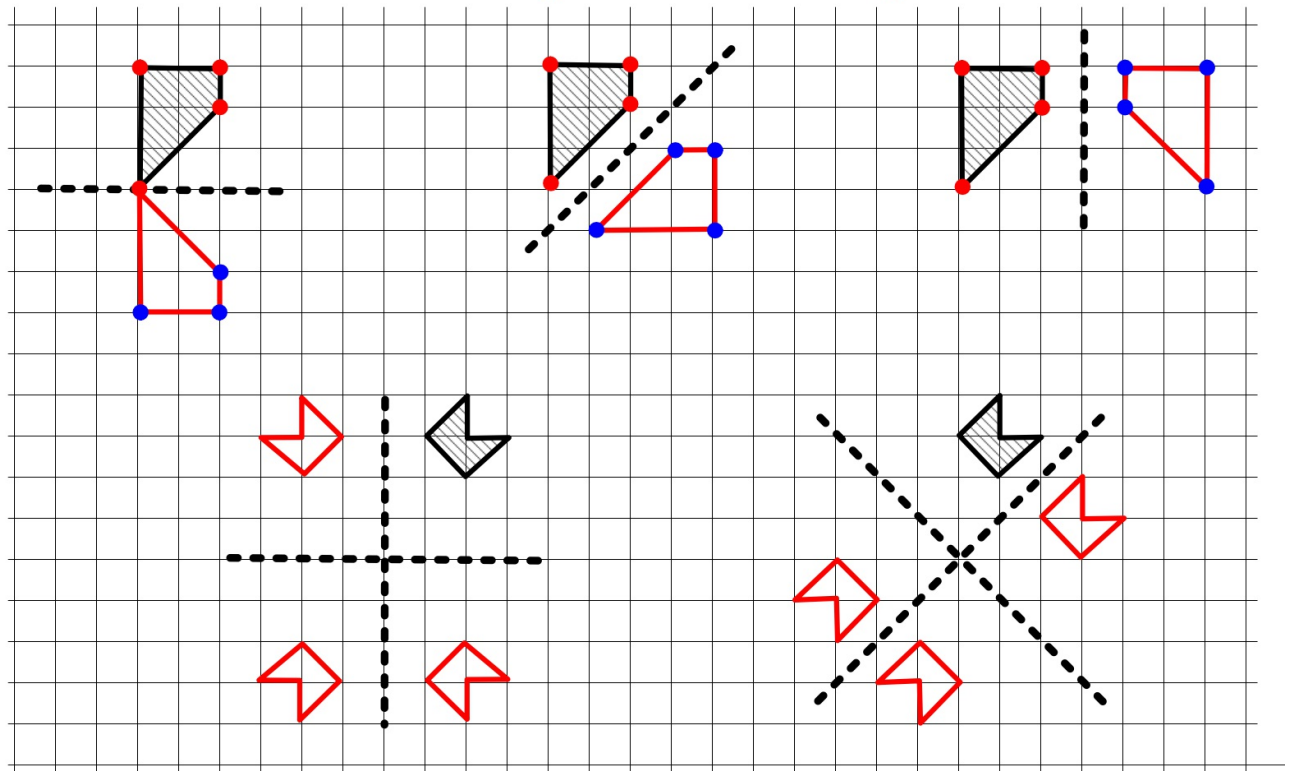
ORDER OF ROTATIONAL SYMMETRY

The number of times a shape fits itself under one turn.

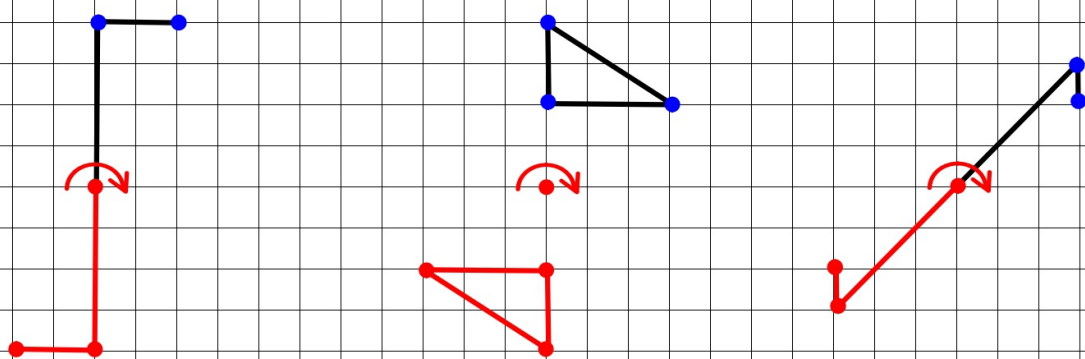


Line symmetry

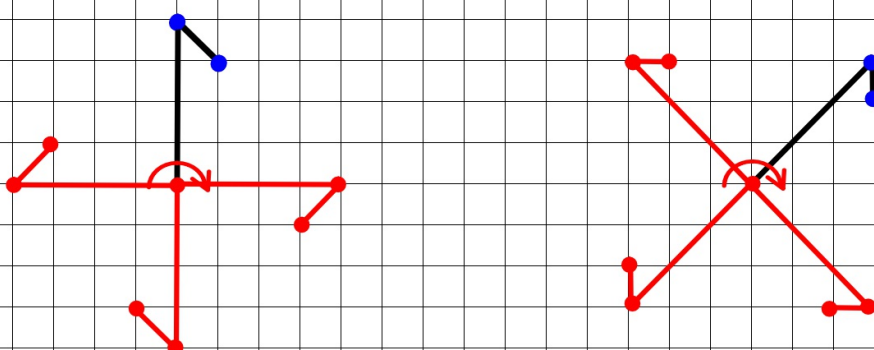
Reflect the corners and join for the image.



Half-turn symmetry

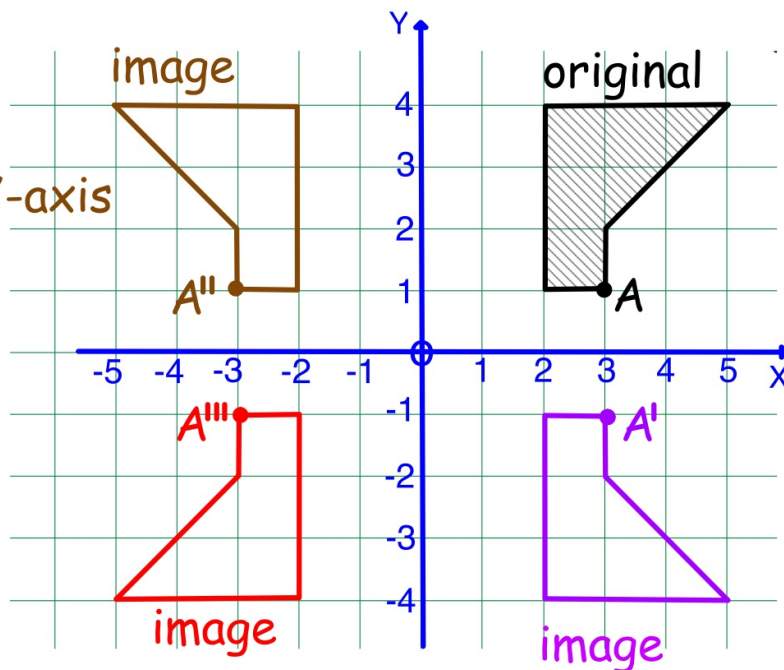


Quarter-turn symmetry



COORDINATES

reflection in the Y-axis

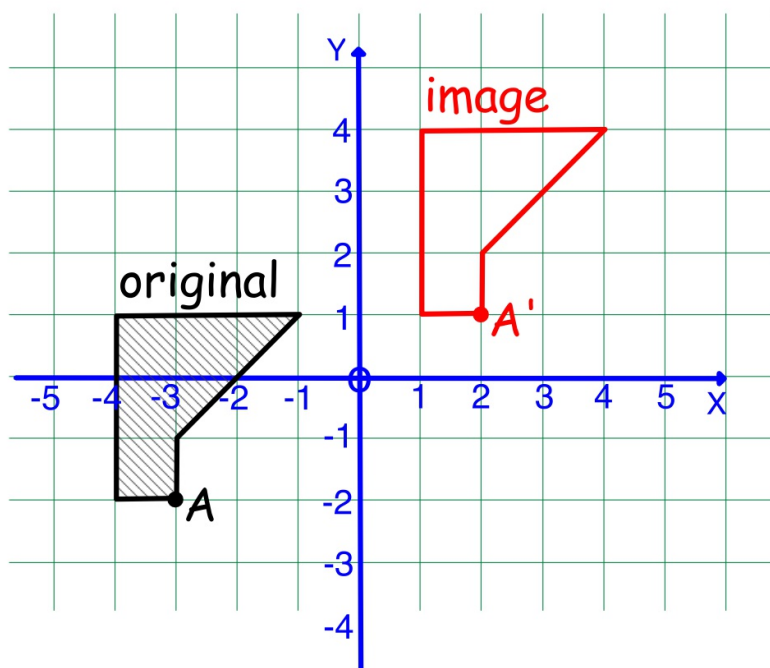


half-turn about O

reflection in the X-axis

image under $A(-3,-2) \rightarrow A'(2,1)$

all points move
5 right, 3 up



ENLARGEMENT and REDUCTION

Angles are unchanged.

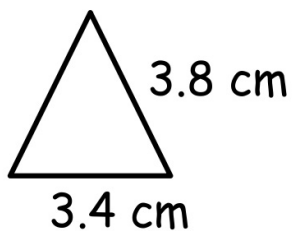
Sides are enlarged/reduced by a SCALE FACTOR.

ENLARGEMENT: $SF > 1$

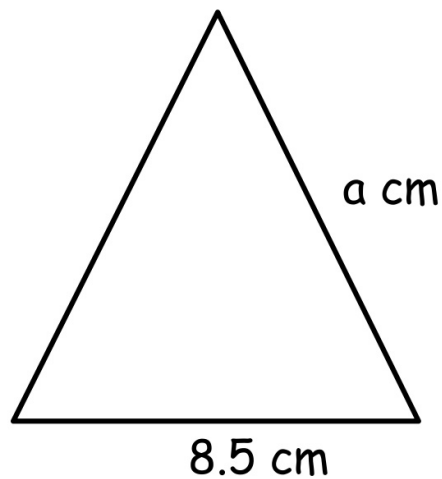
REDUCTION: $0 < SF < 1$

The shape with the dimension to be found is the image.

$$\text{Scale Factor} = \frac{\text{image size}}{\text{original size}}$$



original



image

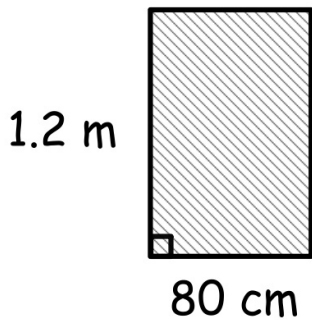
$$SF = \frac{8.5}{3.4} = 2.5$$

$$a = 3.8 \times 2.5 = 9.5$$

CHAPTER 11: AREA

RECTANGLE $A = lb$

match length units to required area units



$$\begin{aligned} A &= lb \\ &= 1.2 \times 0.8 \\ &= 0.96 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A &= lb \\ &= 120 \times 80 \\ &= 9600 \text{ cm}^2 \end{aligned}$$

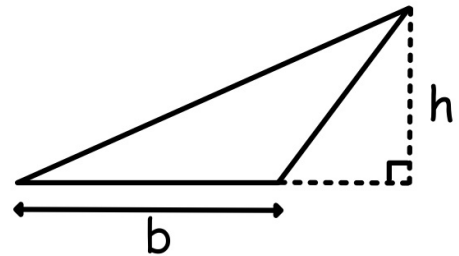
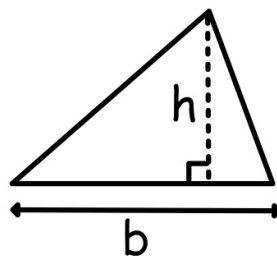
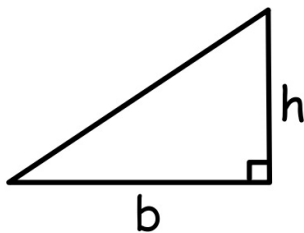
TRIANGLES

$$A = \frac{1}{2} bh$$

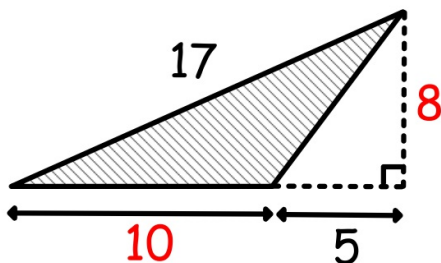
The height and the base are at 90° .

(altitude)

(perpendicular)



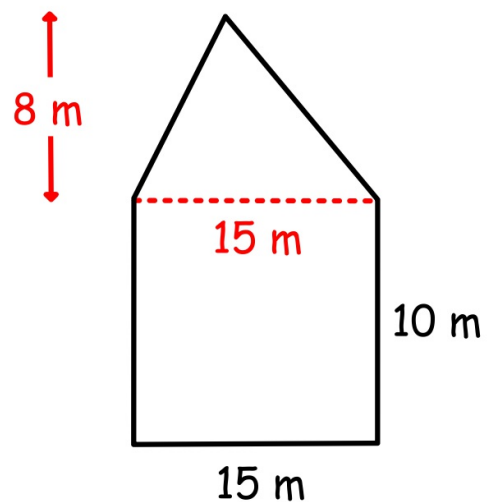
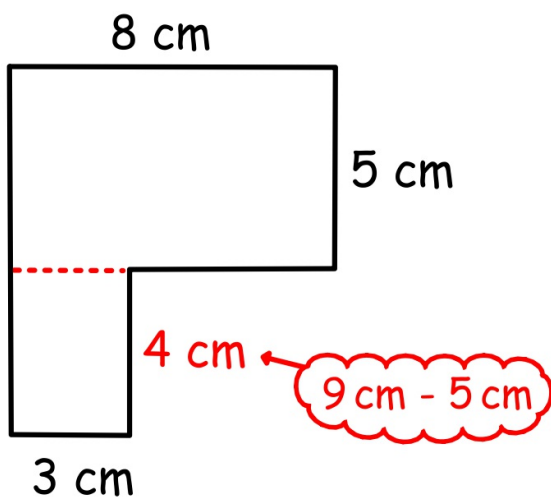
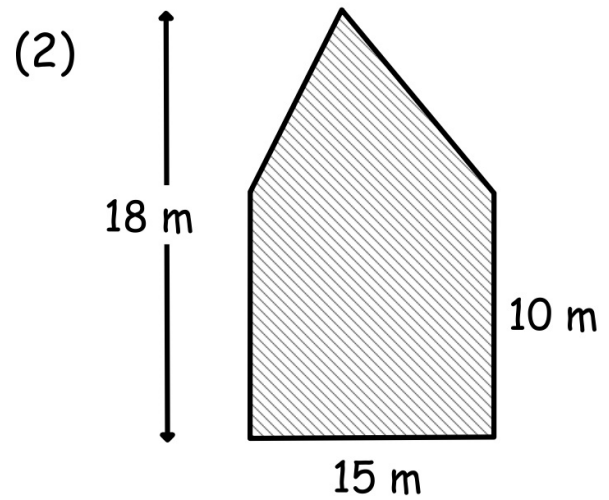
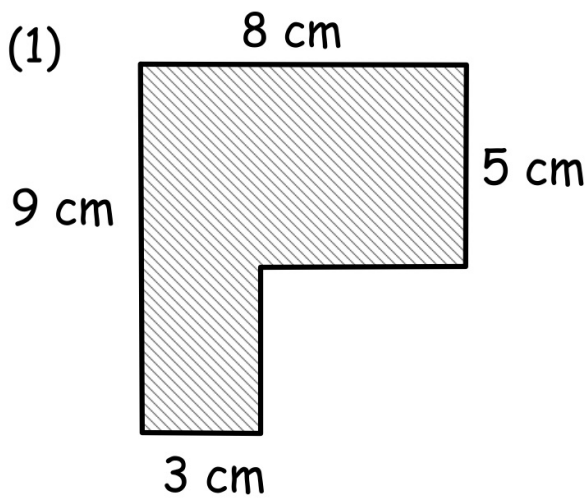
use base and height - ignore extra numbers



$$\begin{aligned} A &= \frac{1}{2} bh \\ &= 10 \times 8 \div 2 \\ &= 40 \text{ units}^2 \end{aligned}$$

COMPOSITE SHAPES

Formed from rectangles and triangles



$$\begin{aligned}
 A &= lb & A &= lb \\
 &= 8 \times 5 & &= 4 \times 3 \\
 &= 40 \text{ cm}^2 & &= 12 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 &40 \text{ cm}^2 + 12 \text{ cm}^2 \\
 &= 52 \text{ cm}^2
 \end{aligned}$$

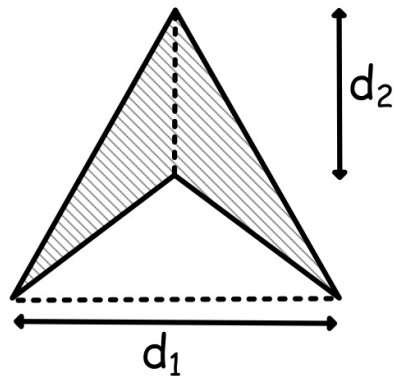
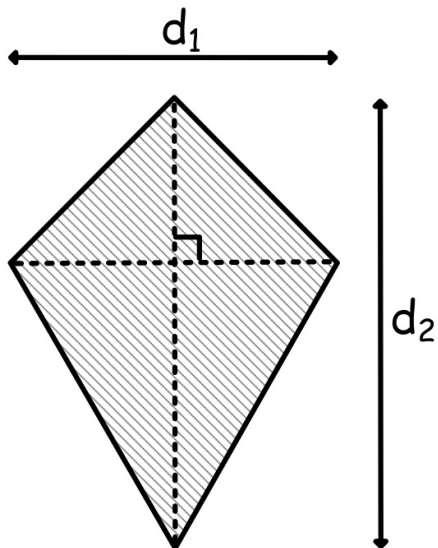
$$\begin{aligned}
 A &= lb & A &= \frac{1}{2} bh \\
 &= 15 \times 10 & &= 15 \times 8 \div 2 \\
 &= 150 \text{ m}^2 & &= 60 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 &150 \text{ m}^2 + 60 \text{ m}^2 \\
 &= 210 \text{ m}^2
 \end{aligned}$$

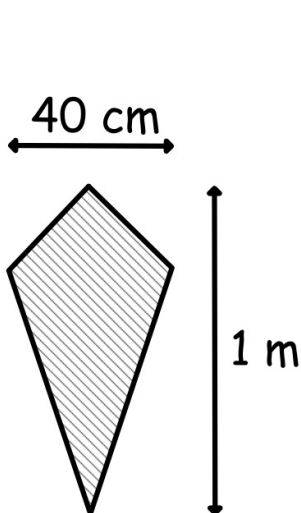
KITE and RHOMBUS

$A = \frac{1}{2}$ the product of the diagonals

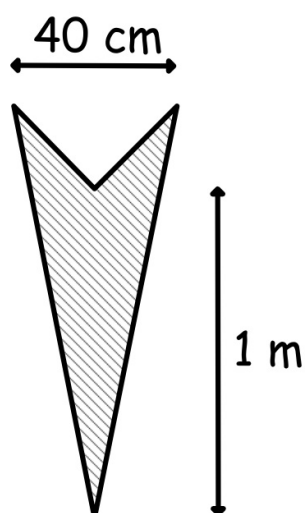
$$A = \frac{1}{2} d_1 d_2$$



Ensure the units match.



$$\begin{aligned} A &= \frac{1}{2} d_1 d_2 \\ &= 40 \times 100 \div 2 \\ &= \underline{\underline{2000\text{ cm}^2}} \end{aligned}$$

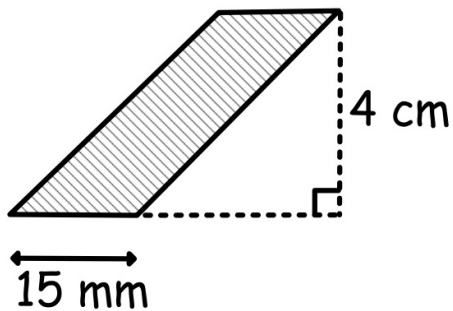
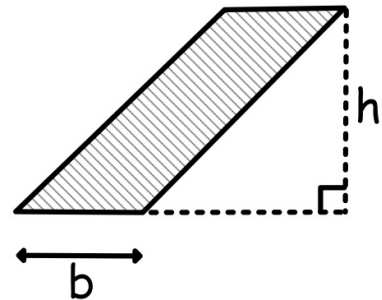
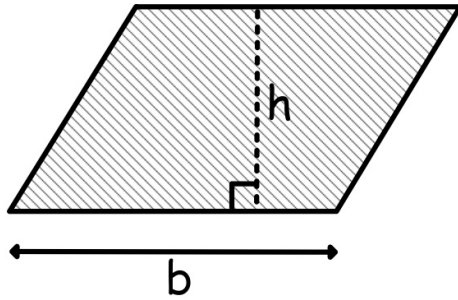


or

$$\begin{aligned} A &= \frac{1}{2} d_1 d_2 \\ &= 0.4 \times 1 \div 2 \\ &= \underline{\underline{0.2\text{ m}^2}} \end{aligned}$$

PARALLELOGRAM

$$A = bh$$

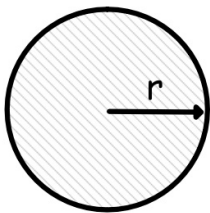


$$\begin{aligned} A &= bh \\ &= 1.5 \times 4 \\ &= \underline{\underline{6 \text{ cm}^2}} \end{aligned}$$

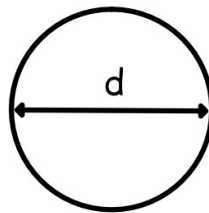
$$\begin{aligned} \text{or } A &= bh \\ &= 15 \times 40 \\ &= \underline{\underline{600 \text{ mm}^2}} \end{aligned}$$

CIRCLES

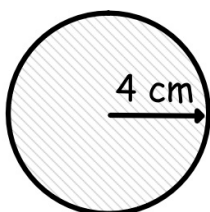
AREA: $A = \pi r^2$



CIRCUMFERENCE: $C = \pi d$



Remember $r = \frac{1}{2} d$ and $\pi \approx 3.14$

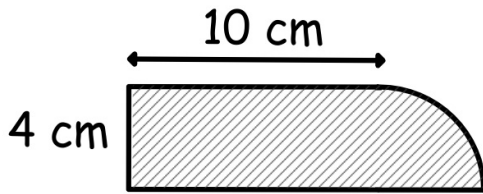


$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times 4 \times 4 \\ &= 50.24 \text{ cm}^2 \end{aligned}$$

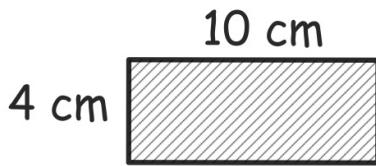
$$\begin{aligned} C &= \pi d \\ &= 3.14 \times 8 \\ &= 25.12 \text{ cm} \end{aligned}$$

COMPOSITE SHAPES

Identify the rectangle and circle parts.



AREA:



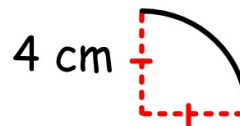
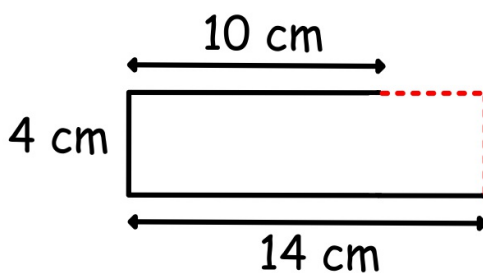
$$\begin{aligned} A &= lb \\ &= 4 \times 10 \\ &= 40 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times 4 \times 4 \\ &= 50.24 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A &= 50.24 \div 4 \\ &= 12.56 \text{ cm}^2 \end{aligned}$$

$$\text{Total Area} = 12.56 + 40 = \underline{\underline{52.56 \text{ cm}^2}}$$

PERIMETER:



$$\begin{aligned} C &= \pi d \\ &= 3.14 \times 8 \\ &= 25.12 \text{ cm} \end{aligned}$$

$$\begin{aligned} &25.12 \div 4 \\ &= 6.28 \text{ cm} \end{aligned}$$

$$\text{Perimeter} = 4 + 10 + 14 + 6.28 = \underline{\underline{34.28 \text{ cm}}}$$