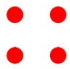
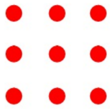
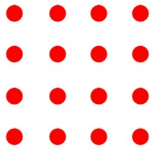
CHAPTER 12: SEQUENCES

square numbers 0, 1, 4, 9, 16, 25 ...


1x1
1

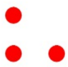

2x2
1+3

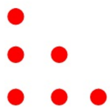

3x3
1+3+5

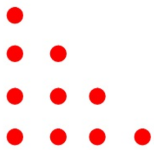

4x4
1+3+5+7

triangular numbers 0, 1, 3, 6, 10, 15 ...


1x2÷2
1


2x3÷2
1+2


3x4÷2
1+2+3


4x5÷2
1+2+3+4

GENERALISE

find (a) a formula for the number of matches.

(b) the number of matches for 10 triangles.

(c) the number of triangles for 51 matches.

(a)

triangles	1	2	3	4	5	T
matches	3	5	7	9	11	2T + 1

add 2

Formula: M = 2T + 1

(b) M = 2x10 + 1 = 21

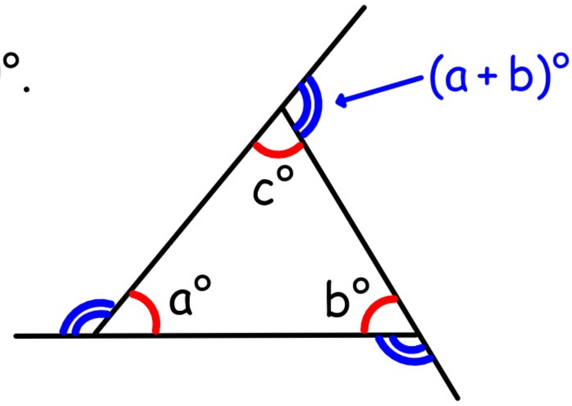
(c) 2T + 1 = 51
2T = 50
T = 25

CHAPTER 13: TRIANGLES

INTERIOR angles add up to 180°.

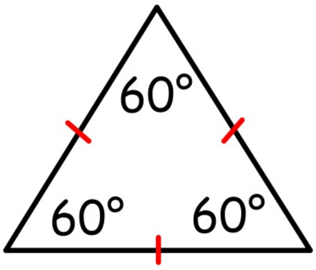
interior angles

exterior angles



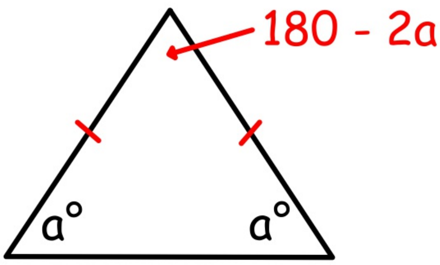
EQUILATERAL TRIANGLES

Three equal sides,
three equal angles of 60°.

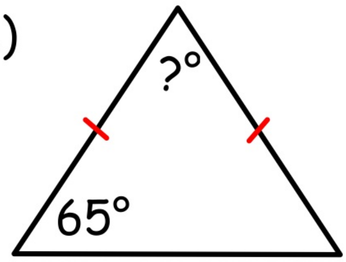


ISOSCELES TRIANGLES

Two equal sides, two equal angles.

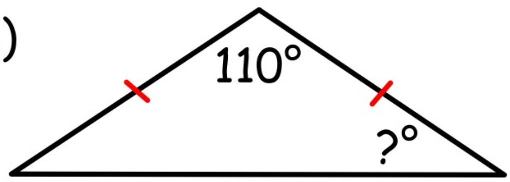


(1)



$$65 + 65 = 130 \qquad 180 - 130 = 50$$

(2)



$$180 - 110 = 70 \qquad 70 \div 2 = 35$$

CHAPTER 14: RATIO and PROPORTION

RATIO

Comparing the sizes of quantities, so there are no units.

school trip: ratio of teachers to pupils is 1:10

model ship: scale 1:400

TV screen: width to height 4:3 (aspect ratio)

Fully simplify ratios: $a : b$

$= a \div HCF : b \div HCF$

(1) 8 : 12 divide by 4

= 2 : 3

(2) 6 kg : 900 g same units

= 6000 g : 900 g divide by 100

= 60 : 9 divide by 3

= 20 : 3

UNITARY RATIO

Express 5 : 4 in the form (a) 1 : n (b) n : 1

(a) 5 : 4 divide by 5

= 1 : 0.8

(b) 5 : 4 divide by 4

= 1.25 : 1

Can multiply up ratios: $a : b$

$= a \times n : b \times n$

Mix yellow and blue paint in the ratio 2 : 3
How much blue paint for 8 tins of yellow ?

Y : B yellow tins: 8 ÷ 2 = 4

= 2 : 3 multiply by 4

= 8 : 12

12 tins of blue

SHARING

Tim and Tom buy 60 chocolates.
Tim contributes £3 and Tom £2.
How many chocolates should each get ?

share 60 chocs in ratio 3:2

number of shares	3 + 2 = 5 shares
one share	60 ÷ 5 = 12 chocs
Tim 3 shares	3 x 12 = 36 chocs
Tom 2 shares	2 x 12 = 24 chocs

DIRECT PROPORTION:

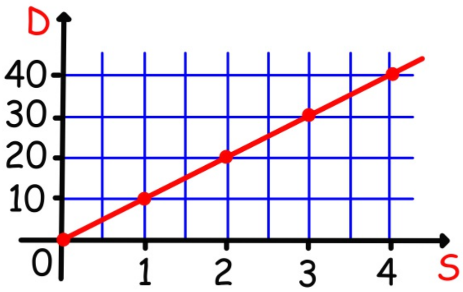
DISTANCE and SPEED are directly proportional

2 x speed results in 2 x distance travelled

One quantity is a multiple of the other.

GRAPH: a straight line through the origin

speed (S)	0	1	2	3	4
distance (D)	0	10	20	30	40



$D = 10 \times S$

Ten books cost £36.

(a) Find the cost of seven books.

(b) How many books for £54 ?

Rate: £3.60 per book

(a) 10 books ⟶ £36

1 book ⟶ £36 ÷ 10

7 books ⟶ £36 ÷ 10 x 7 = **£25.20**

(b) £36 ⟶ 10 books

£1 ⟶ 10 ÷ 36 books

£54 ⟶ 10 ÷ 36 x 54 = **15 books**

or £54 ÷ £3.60

INVERSE PROPORTION:

TIME and SPEED are inversely proportional

2 x speed results in $^1/_2$ x time taken

The quantities multiply to the same product.

speed (S)	1	2	3	4
time (T)	12	6	4	3

$S \times T = 12$

A school has money to buy 50 books at £18 each.
Price increases to £20. How many books can be bought?

50 books at £18 each total money = 50 x £18 = £900

£900 at £20 each Nº books = 900 ÷ 20 = 45 books

OR

18 £/book ⟶ 50 books

1 £/book ⟶ 50 x 18 (900 books)

20 £/book ⟶ 50 x 18 ÷ 20 = 45 books

CHAPTER 15: FRACTIONS and PERCENTAGES

1%	5%	10%	20%	25%
$^1/_{100}$	$^1/_{20}$	$^1/_{10}$	$^1/_5$	$^1/_4$

$33^1/_3$%	50%	$66^2/_3$%	75%
$^1/_3$	$^1/_2$	$^2/_3$	$^3/_4$

(1) Find 4% of 250 kg

non-calculator:

1% 250 ÷ 100 = 2.5 kg

4% 2.5 x 4 = 10 kg

by calculator:

250 ÷ 100 x 4

= 10 kg

(2) Find $66^2/_3$% of 18 m

$^2/_3$ of 18 m

18 ÷ 3 = 6 m

6 x 2 = 12 m

(3) Find 35% of £240

non-calculator:

10% £240 ÷ 10 = £24

30% £24 x 3 = £72

5% £24 ÷ 2 = £12

£84

by calculator:

£240 ÷ 100 x 35

= £84

EXPRESSING CHANGE AS A PERCENTAGE

$$\% \text{ CHANGE} = \frac{\text{CHANGE}}{\text{START VALUE}} \times 100\%$$

A £15000 car is resold for £12000. Find the % loss.

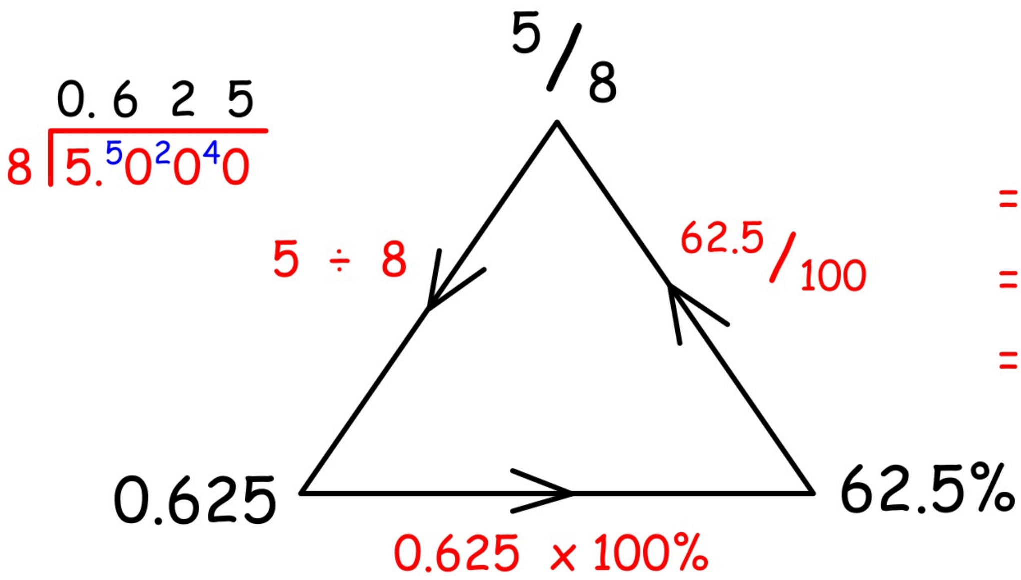
actual loss = £15000 - £12000 = £3000

$$\% \text{ loss} = \frac{\text{loss}}{\text{start value}} \times 100\%$$

$$= \frac{£3000}{£15000} \times 100\%$$

$$= 20\%$$

SWITCHING BETWEEN FORMS



$$\frac{625}{1000} = \frac{125}{200} = \frac{25}{40} = \frac{5}{8}$$

EQUAL FRACTIONS

$$\frac{3}{4} = \frac{3\times 6}{4\times 6} = \frac{18}{24}$$

SIMPLIFYING:

$$\frac{18}{24} = \frac{18\div 6}{24\div 6} = \frac{3}{4}$$

MIXED NUMBERS

$$2\tfrac{3}{4} = 2 + \tfrac{3}{4} = \tfrac{8}{4} + \tfrac{3}{4} = \tfrac{11}{4}$$

$$2\tfrac{3}{4} = \tfrac{11}{4}$$

$4 \times 2 + 3 = 11$

$$\tfrac{11}{4} = \tfrac{8}{4} + \tfrac{3}{4} = 2 + \tfrac{3}{4} = 2\tfrac{3}{4}$$

$$\tfrac{11}{4} = 2\tfrac{3}{4}$$

$11 \div 4 = 2 \text{ R } 3$

ADD and SUBTRACT

requires a common denominator, the LCM (least common multiple)

(1) $\tfrac{2}{5} + \tfrac{3}{10}$

$= \tfrac{4}{10} + \tfrac{3}{10}$

$= \tfrac{7}{10}$

(2) $\tfrac{5}{6} - \tfrac{2}{9}$

$= \tfrac{15}{18} - \tfrac{4}{18}$

$= \tfrac{11}{18}$

for mixed numbers treat fractions and whole numbers separately.

(3) $12\tfrac{5}{6} - 3\tfrac{2}{9}$

$= 12\tfrac{15}{18} - 3\tfrac{4}{18}$

$= 9\tfrac{11}{18}$

MULTIPLY $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

"of" means x

change mixed numbers to "top-heavy" fractions

(1) $^{3}/_{10}$ of $2\,^{3}/_{4}$

$= \,^{3}/_{10} \times \,^{11}/_{4}$

$= \,^{33}/_{40}$

(2) $1\,^{2}/_{3} \times 3\,^{1}/_{5}$

$= \,^{5}/_{3} \times \,^{16}/_{5}$

$= \,^{80}/_{15}$

$= \,^{16}/_{3}$

$= 5\,^{1}/_{3}$

DIVIDE $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

RECIPROCAL

(1) $^{5}/_{6} \div \,^{3}/_{7}$

$= \,^{5}/_{6} \times \,^{7}/_{3}$

$= \,^{35}/_{18}$

$= 1\,^{17}/_{18}$

(2) $1\,^{2}/_{7} \div 4$

$= \,^{9}/_{7} \div \,^{4}/_{1}$

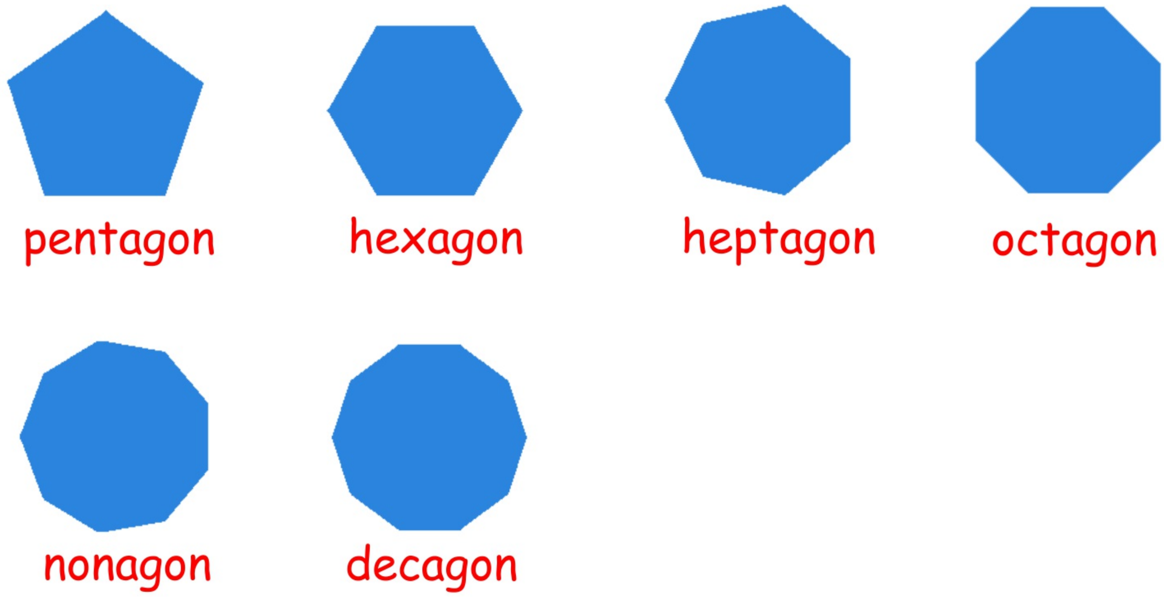
$= \,^{9}/_{7} \times \,^{1}/_{4}$

$= \,^{9}/_{28}$

CHAPTER 16: 2D SHAPES

POLYGON a many sided shape.

In a REGULAR polygon all sides and angles are equal.



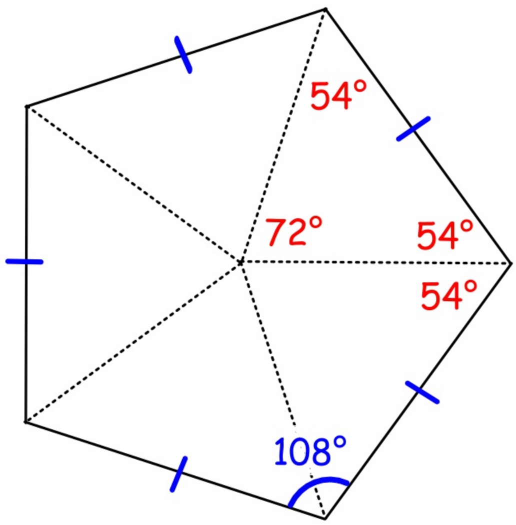
REGULAR PENTAGON

angle at the centre $360° \div 5 = 72°$

isosceles Δ $180° - 72° = 108°$

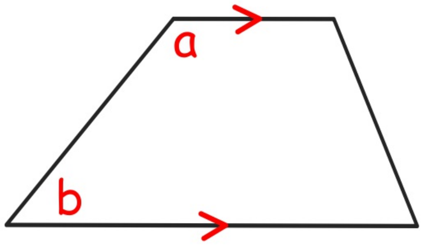
$108° \div 2 = 54°$

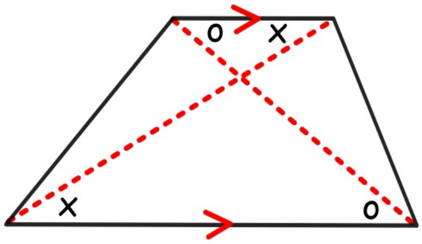
interior angle $54° \times 2 = 108°$



QUADRILATERALS: angle sum 360°

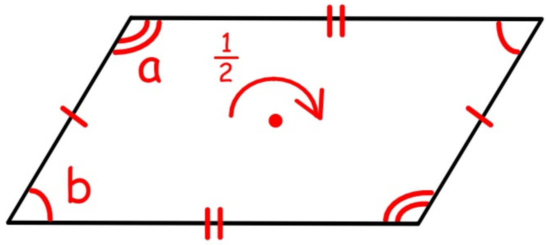
TRAPEZIUM:

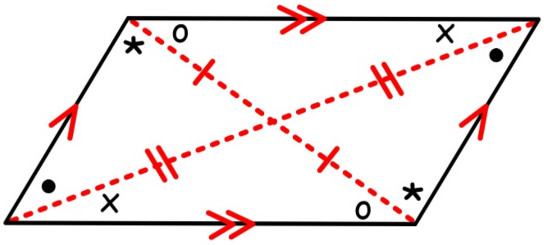




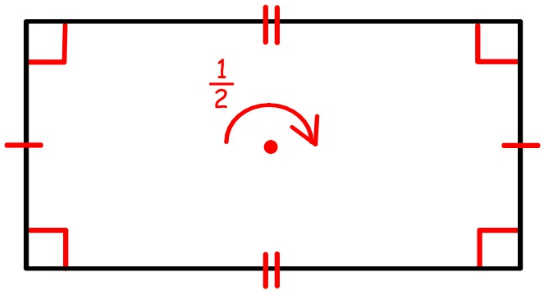
$a + b = 180°$

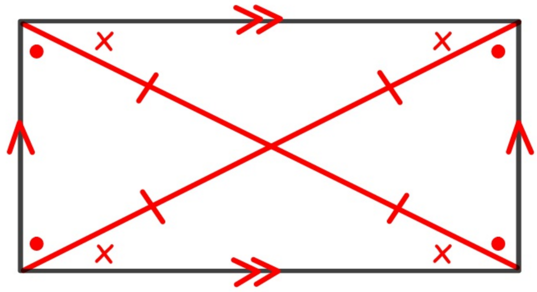
PARALLELOGRAM: a trapezium



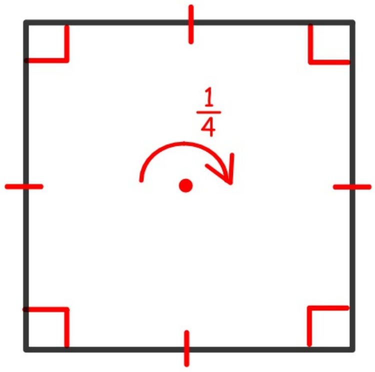


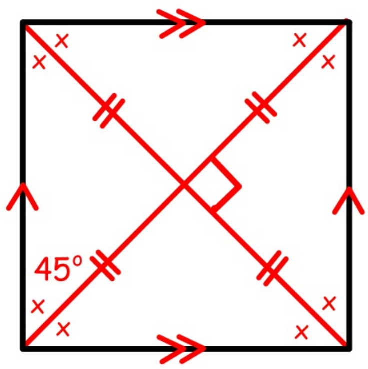
RECTANGLE: a parallelogram



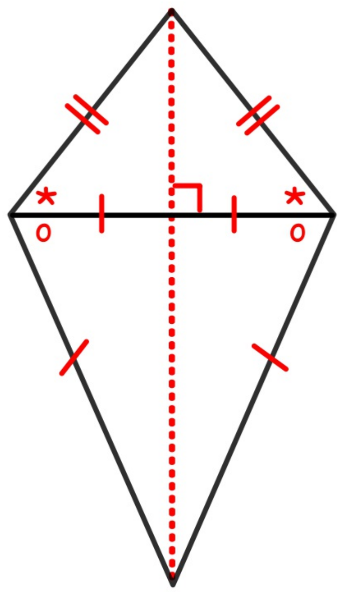


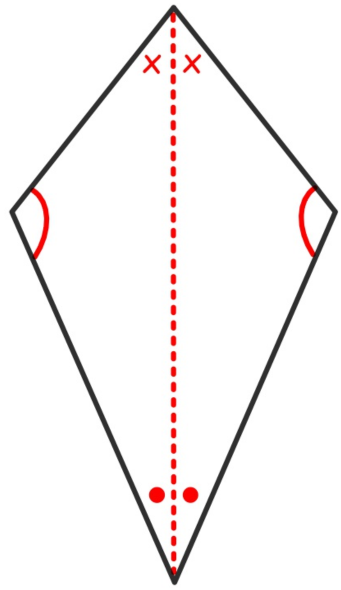
SQUARE: a rectangle



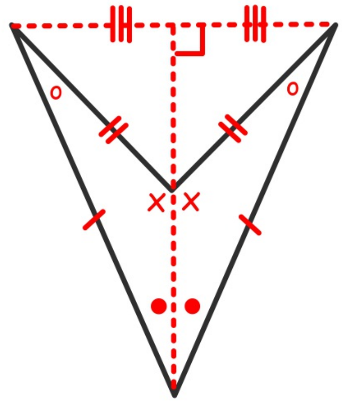


KITE

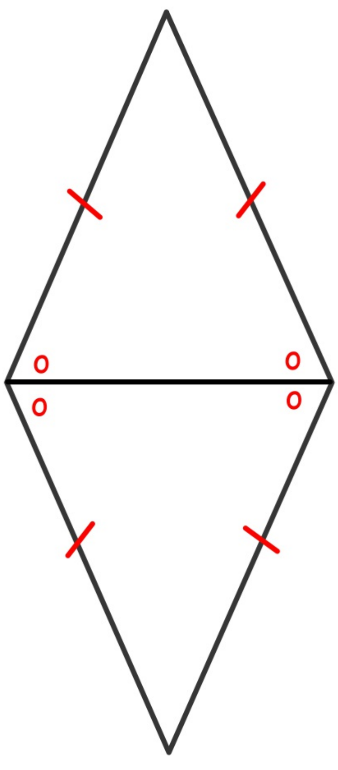


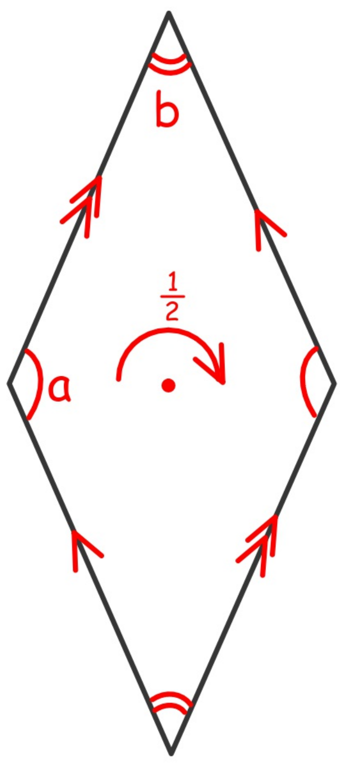


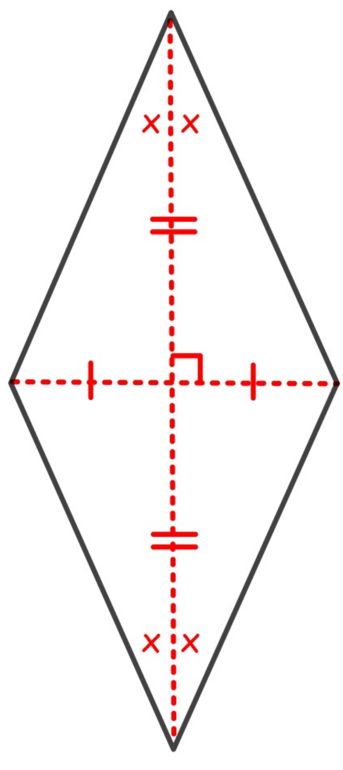
V-KITE



RHOMBUS   a kite and a parallelogram



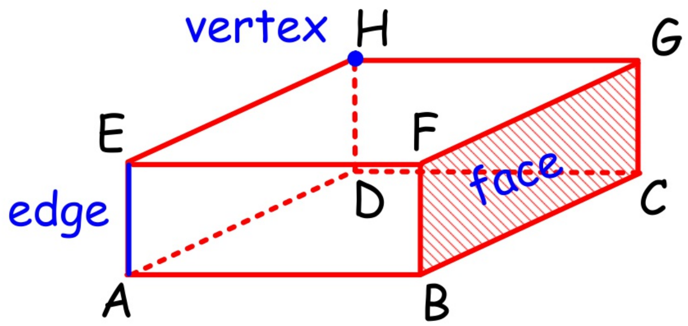




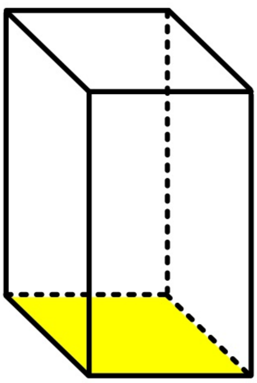
CHAPTER 17: 3D SHAPES

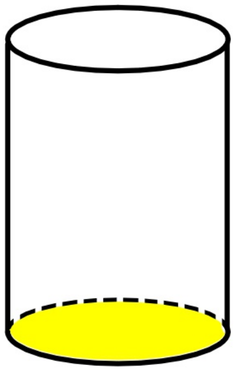
PRISM — A 3D solid which has the same cross-section throughout its length.

PYRAMID — The cross-section reduces to a point.

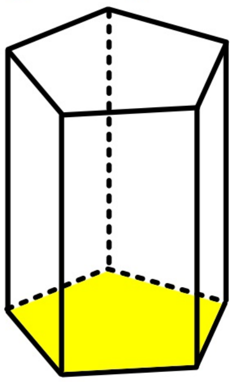


vertex H
edge AE
face BCGF
(not BCFG)

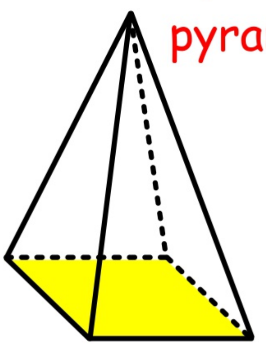


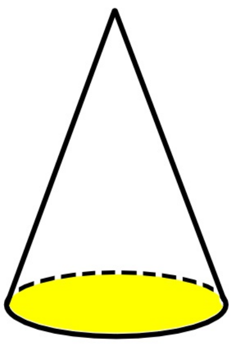


pentagonal prism

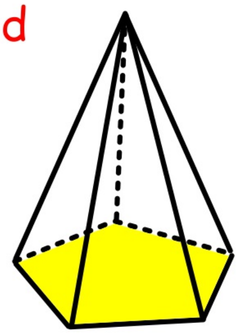


square-based pyramid





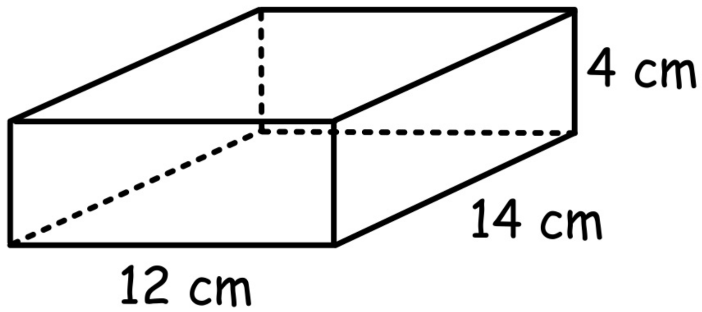
pentagonal-based pyramid



SKELETON MODEL

Shows edges - 'hidden' edges dotted.

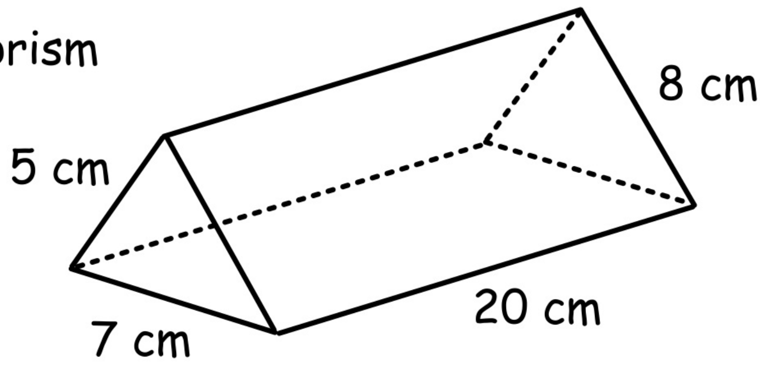
(1) cuboid



total length of edge:

$$\begin{array}{r} 12 \\ 14 \\ +\ 4 \\ \hline 30 \end{array} \times 4 = \underline{\underline{120\ \text{cm}}}$$

(2) triangular prism



total length of edge:

$2 \times 5 = 10$

$2 \times 7 = 14$

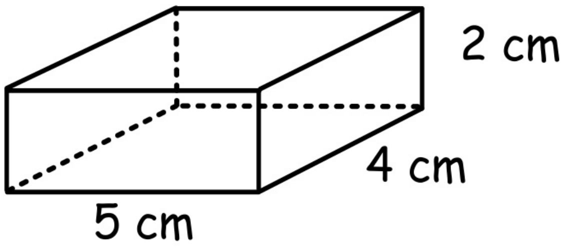
$2 \times 8 = 16$

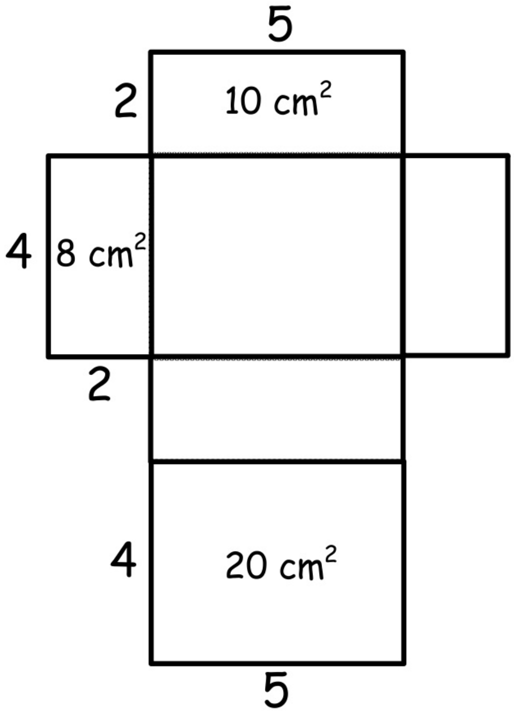
$3 \times 20 = 60$

$\underline{\underline{100}}$ cm

NET Flattened out solid, showing the connected faces.

(1) cuboid





total surface area:

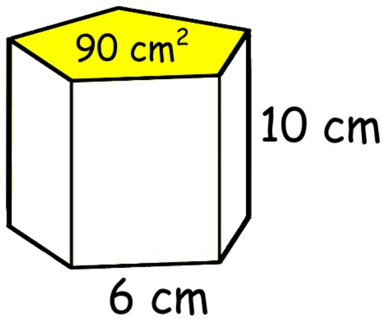
$5 \times 4 = 20\ cm^2$

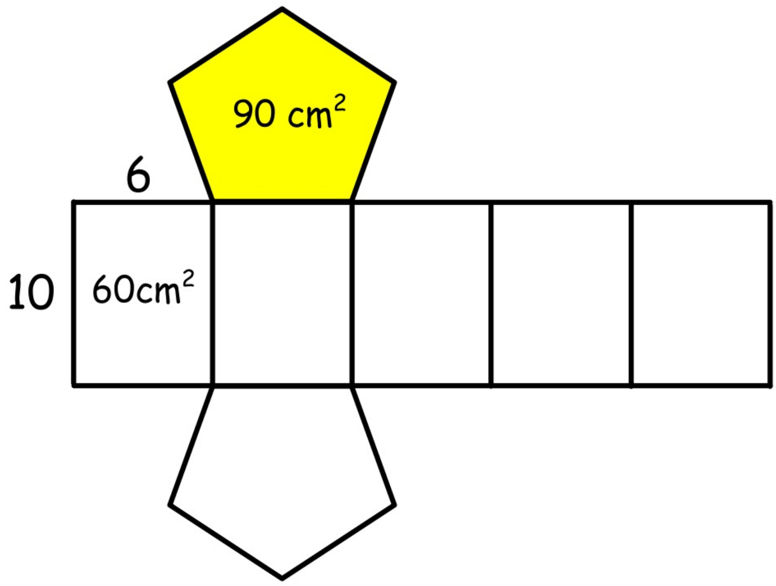
$5 \times 2 = 10\ cm^2$

$4 \times 2 = 8\ cm^2$

$38\ cm^2 \times 2 = \underline{\underline{76\ cm^2}}$

(2) pentagonal prism





total surface area:

$10 \times 6 = 60\ cm^2$

$5 \times 60\ cm^2 = 300\ cm^2$

$2 \times 90\ cm^2 = 180\ cm^2$

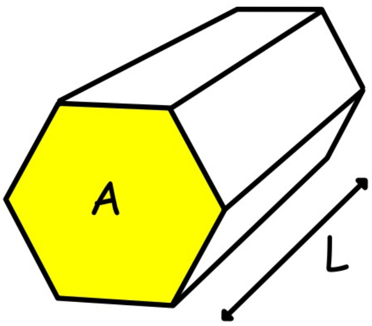
total $\underline{\underline{480\ cm^2}}$

VOLUME OF PRISMS and PYRAMIDS

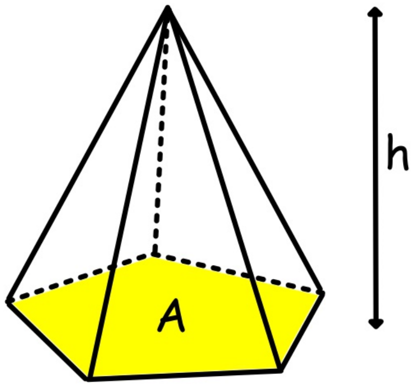
ensure length units match area units eg. cm with cm^2

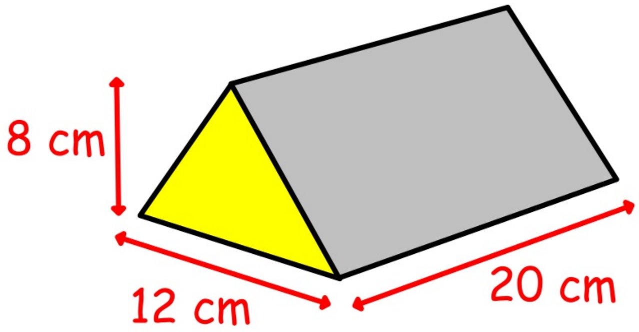
PRISM $V = AL$

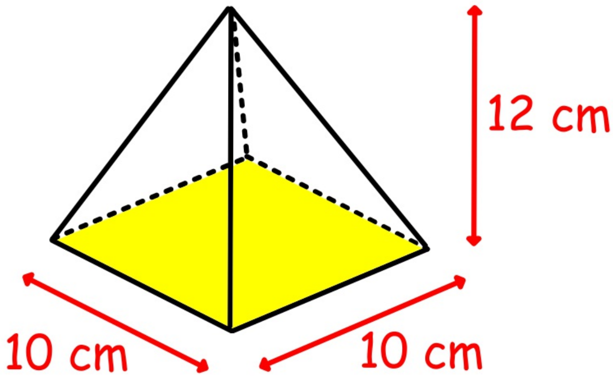
(or $V = Ah$)



PYRAMID $V = \frac{1}{3} A h$







$A = {}^1/_2\, bh$

$= 12 \times 8 \div 2$

$= 48\ cm^2$

$V = AL$

$= 48 \times 20$

$= 960\ cm^3$

$A = lb$

$= 10 \times 10$

$= 100\ cm^2$

$V = \frac{1}{3} Ah$

$= 100 \times 12 \div 3$

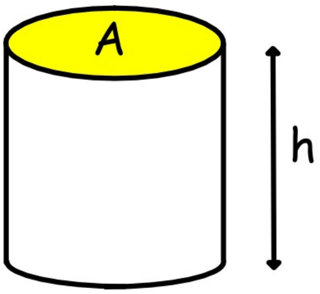
$= 400\ cm^3$

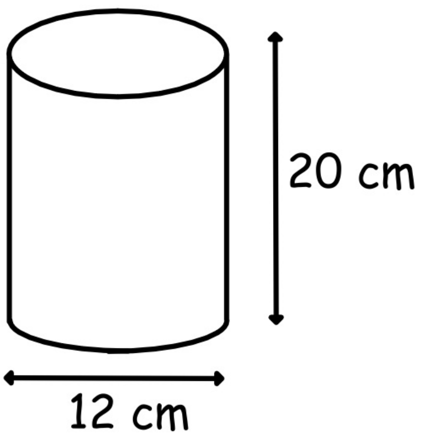
CYLINDER and CONE

Ensure radius and height units match. eg. cm with cm

$V = Ah$ and $A = \pi r^2$

$$V = \pi r^2 h$$





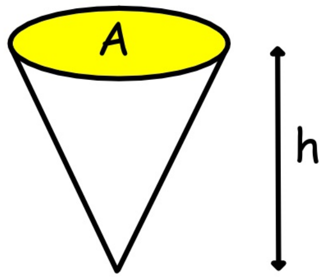
$V = \pi r^2 h$

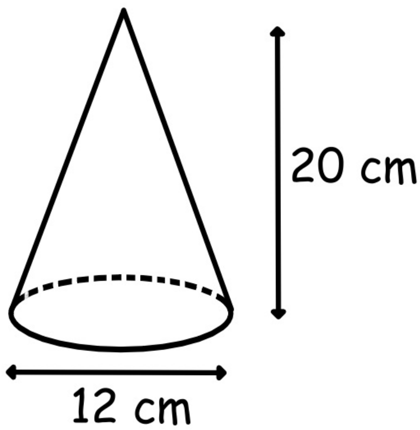
$= \pi \times 6 \times 6 \times 20$

$= 2261.946...$

$\approx \underline{\underline{2260 \text{ cm}^3}}$

$$V = \frac{1}{3}\pi r^2 h$$





$V = \frac{1}{3}\pi r^2 h$

$= \pi \times 6 \times 6 \times 20 \div 3$

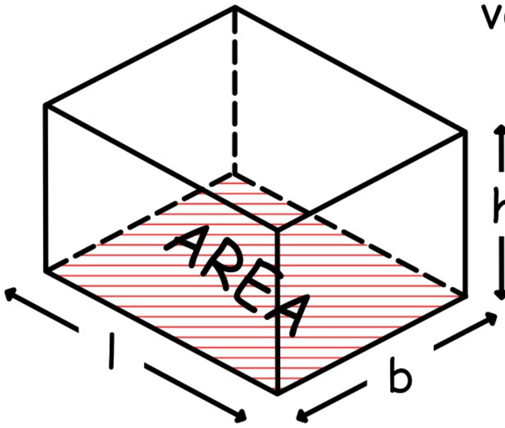
$= 753.982...$

$= \underline{\underline{754 \text{ cm}^3}}$

CUBOID

volume = length × breadth × height

volume = area of base × height

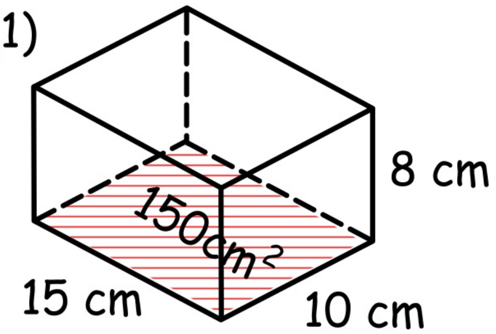


1000 cm^3 = 1 litre

1000 ml = 1 litre

1 ml = 1 cm^3

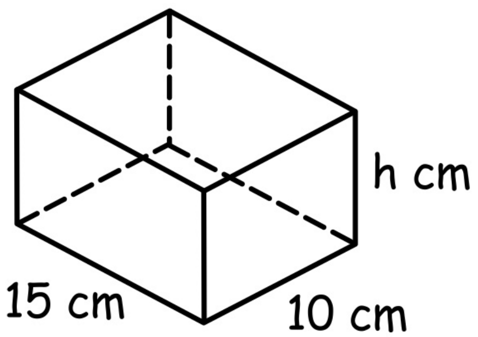
(1)



V = lbh
= 15 × 10 × 8
= 1200 cm^3
= 1.2 litres

V = Ah
= 150 × 8
= 1200 cm^3

(2) volume 1.2 litres



V = lbh
1200 = 15 × 10 × h
1200 = 150 × h
h = 1200 ÷ 150
h = 8

CHAPTER 18: INFORMATION HANDLING 2

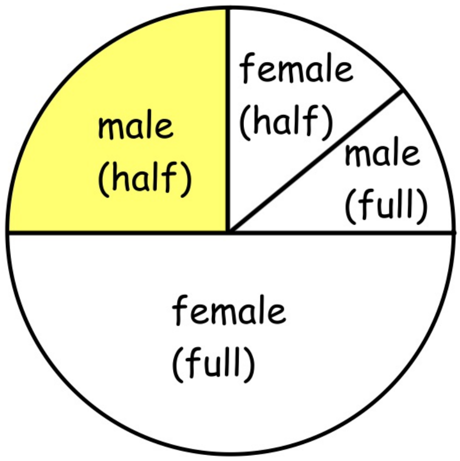
CONTINUOUS DATA can take any value

DISCRETE DATA takes particular values

STATISTICAL DIAGRAMS

read information , make comparisons , identify trends

1800 tickets sold.



male(half)

$^{90}/_{360} \times 1800 = 450$ tickets

Month	Jan	Feb	Mar	April	May
Sales (£1000s)	20	40	30	10	20

PIE CHART

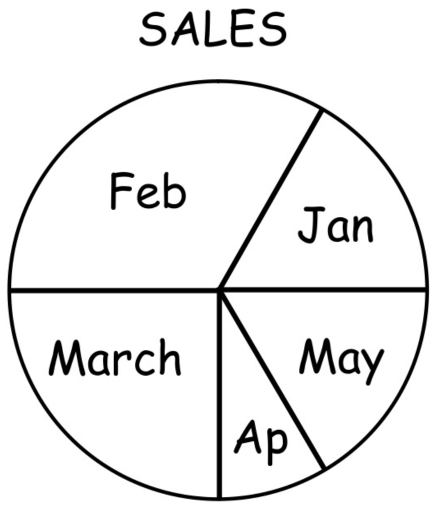
TOTAL SALES = 120

Jan,May $\frac{20}{120} \times 360° = 60°$

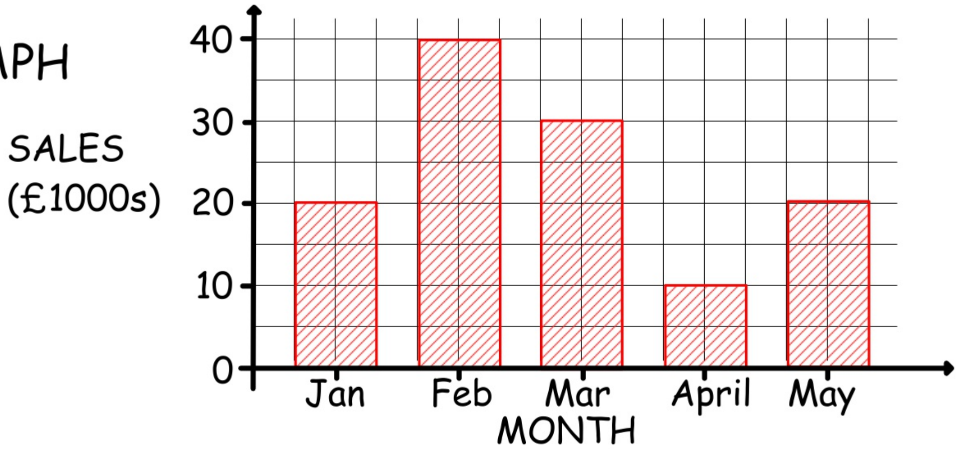
Feb $\frac{40}{120} \times 360° = 120°$

March $\frac{30}{120} \times 360° = 90°$

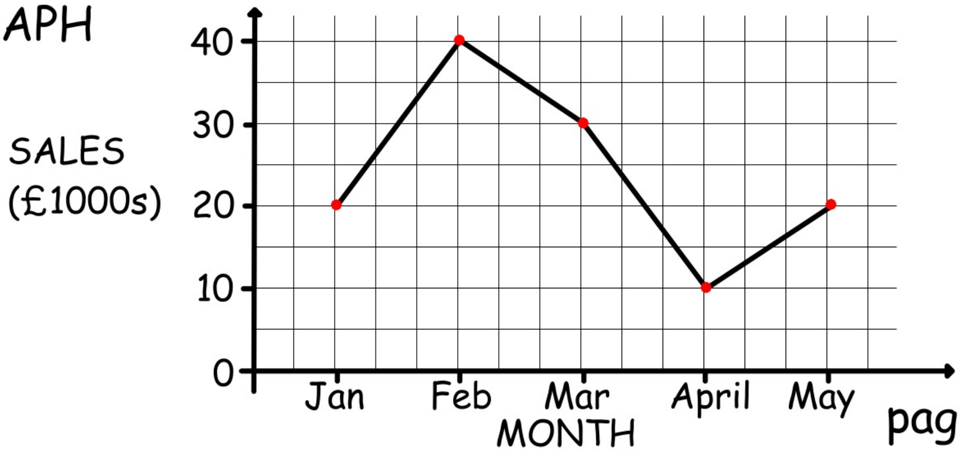
April $\frac{10}{120} \times 360° = 30°$



BAR GRAPH



LINE GRAPH



STEM-AND-LEAF DIAGRAM

Prepare unordered first

5.6 , 3.9 , 6.4 , 4.5 , 3.8 , 5.3 , 6.7 , 3.9 , 5.5 , 4.8 ,
5.0 , 5.8 , 6.2 , 4.2 , 6.1 , 5.3 , 4.9 , 7.3 , 4.4

unordered

3	9 8 9
4	5 8 2 9 4
5	6 3 5 0 8 3
6	4 7 2 1
7	3

ordered

3	8 9 9
4	2 4 5 8 9
5	0 3 3 5 6 8
6	1 2 4 7
7	3

n = 19 3|8 = 3.8

BACK-TO-BACK STEM-AND-LEAF

boys

15 15 21 22 23
25 26 31 33 34
37 39 41 46 46

girls

11 19 22 25 25
29 31 34 36 38
40 46 49 50 50

boys		girls
5 5	1	1 9
6 5 3 2 1	2	2 5 5 9
9 7 4 3 1	3	1 4 6 8
6 6 1	4	0 6 9
	5	0 0

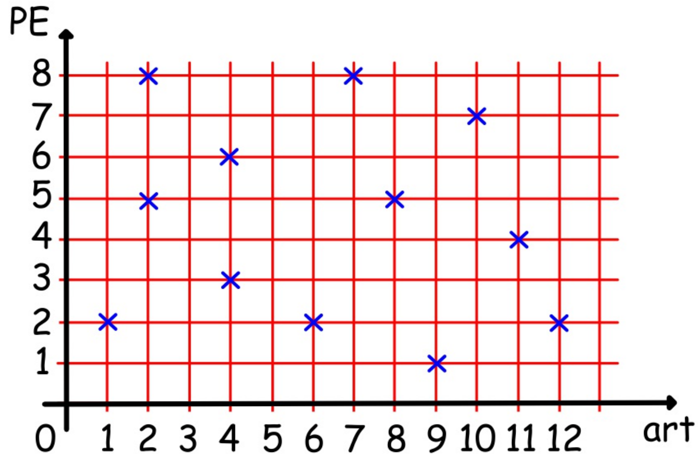
n = 15 n = 15

1|9 = 19

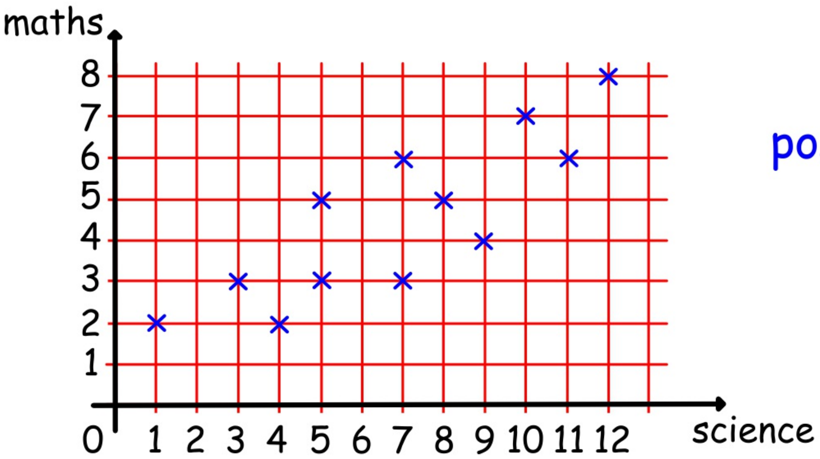
SCATTER DIAGRAMS

If the points plotted lie along a line there is a relationship between the quantities.

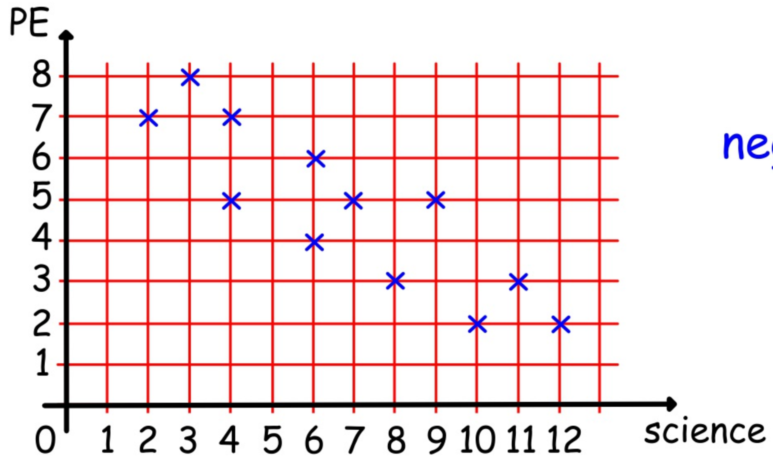
art	2	7	4	8	9	6	11	2	10	1	4	12
PE	8	8	3	5	1	2	4	5	7	2	6	2



no correlation



positive correlation



negative correlation

PROBABILITY

The probability of an event A occurring:

$$P(A) = \frac{\text{number of outcomes involving } A}{\text{number of outcomes possible}}$$

$0 \leq P \leq 1$ P = 0 impossible

P(A) + P(not A) = 1 P = 1 certain

Expected outcomes = P(A) x number of trials

(1) Roll a dice (die) 300 times. How many sixes expected ?

number of sixes = $^1/_6 \times 300 = 50$

(2) choose a letter at random from ARITHMETIC.

$$P(\text{vowel}) = \frac{4}{10} = 0.4$$

$$P(\text{consonant}) = \frac{6}{10} = 0.6 = P(\text{not a vowel})$$

(3) roll two dice , score a total of five.

36 outcomes possible:

(1,1) (1,2) (1,3) ...... (1,6)
(2,1) (2,2) (2,3) ...... (2,6)
. . . .
. . . .
(6,1) (6,2) (6,3) ...... (6,6)

4 outcomes total five:

(1,4) (2,3) (3,2) (4,1)

$$P(\text{five}) = \frac{4}{36} = \frac{1}{9}$$