MATHEMATICS
S1+S2 COURSE NOTES
LEVEL 3/4

I

MATHS

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CHAPTER 1: NUMBER

In Words: place value of figures required.

Th H T U
4 5 7 6 9

forty five thousand, seven hundred and sixty nine.

Reading Scales:

870   940

800   900   1 000

each division 10

2800   5400
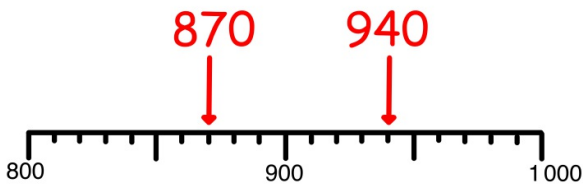
2 000   4 000   6 000
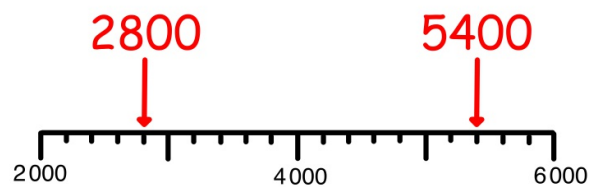
each division 200

CALCULATIONS:

Th H T U
  8 3 7
  9 6 2
+ 5 4 9
  2 3 4 8
  2   1   1

Th H T U
⁸9̸ ¹4 ⁷8̸ ¹3
- 5 7 6 9
  3 7 1 4

    5 3 4
    × 6
  3 2 0 4
  3   2   2

  1 3 5
5 ⌐6 ¹7 ²5

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CALCULATOR:

Read and decide the operation(s) required + , - , x , ÷
Write the calculation(s).

(1) John earns £452 per week. Calculate his annual salary.

£452 x 52 = £23 504

(2) 144 000 paper clips are put into packets of 125.
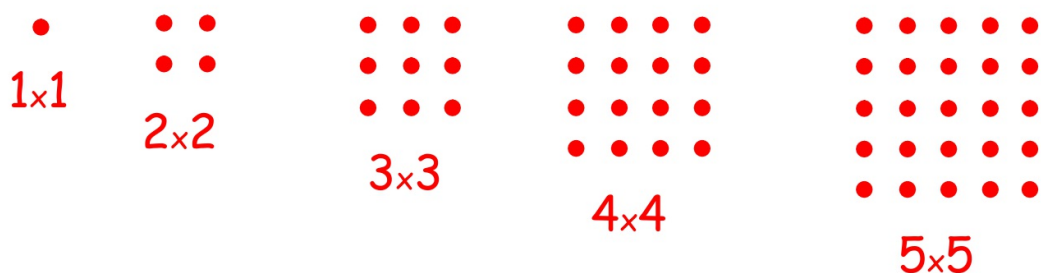How many packets ?   144000 ÷ 125 = 1152

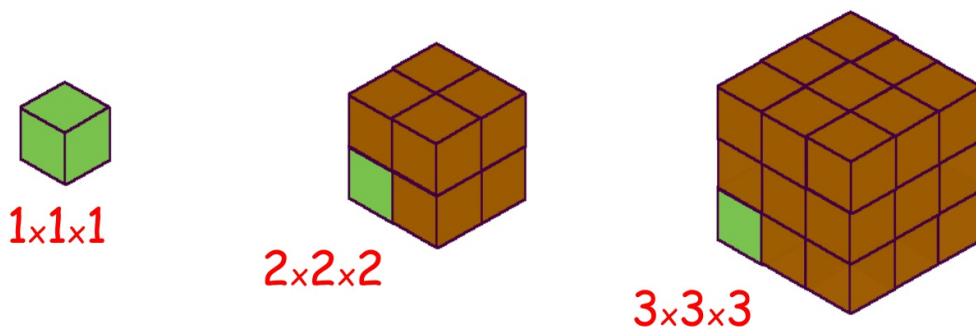(3) A car mileometer changes from 32 647 to 48 312.
How many miles were travelled ?

48312 - 32647 = 15665

NUMBER PATTERNS

square numbers   0, 1, 4, 9, 16, 25 ...

1x1

2x2

3x3

4x4

5x5

cube numbers   0, 1, 8, 27, 64, 125 ...

1x1x1

2x2x2

3x3x3

POWERS

index
base $\to a^n = a \times a \times \ldots \times a$ to n terms

"2 to the power of 5"

(i) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

(ii) $2^7 = \qquad 2^5 \times 2 \times 2 = 32 \times 2 \times 2 = 128$

(i) 2^3

  $= 2 \times 2 \times 2$

  $= 8$

(ii) $(7 - 3)^2$

  $= 4^2$

  $= 4 \times 4$

  $= 16$

(iii) $7^2 - 3^2$

  $= 7 \times 7 - 3 \times 3$

  $= 49 \quad - \quad 9$

  $= 40$

EVALUATE EXPRESSIONS
Replace the letter by its value.

$a = 2$, $b = 3$, $c = 5$

(1) $b^2 - a^3$

  $= b \times b - a \times a \times a$

  $= 3 \times 3 - 2 \times 2 \times 2$

  $= \quad 9 - 8$

  $= \quad 1$

(2) ac^2

  $= a \times c \times c$

  $= 2 \times 5 \times 5$

  $= 2 \times 25$

  $= 50$

(3) $(ac)^2$

  $= (a \times c)^2$

  $= (2 \times 5)^2$

  $= 10^2$

  $= 100$

SQUARE ROOT

Square and square root $3^2 = 9$
are inverse processes. $\sqrt{9} = 3$

ROOTS   1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 ....
SQUARES   1 , 4 , 9 , 16 , 25 , 36 , 49 , 64 .....

approximation: $\sqrt{40}$ lies between 6 and 7.

Solve Equations:

(i) $x^2 = 36$ (ii) $\sqrt{x} = 6$

     $x = \sqrt{36}$ $x = 6^2$

     $x = 6$ $x = 36$

SUM (add)   12 + 4 = 16
DIFFERENCE (subtract)   12 - 4 = 8
PRODUCT (multiply)   12 x 4 = 48
QUOTIENT (divide)   12 ÷ 4 = 3

MULTIPLE: a product by multiplying by an integer.

multiples of 6:   6 , 12 , 18 , 24 ...
multiples of 8:   8 , 16 , 24 , 36 ...
common multiples: 24 , 48 ...
lowest common multiple: 24   LCM = 24

FACTOR: pairs multiply to give the number.

12:        $1 \times 12 = 2 \times 6 = 3 \times 4$

factors of 12:   1 , 2 , 3 , 4 , 6 , 12
factors of 18:   1 , 2 , 3 , 6 , 9 , 18
common factors: 1 , 2 , 3 , 6
highest common factor: 6   HCF = 6

PRIME NUMBER: has exactly 2 factors, itself and 1.

2, 3, 5, 7, 11, 13, 17...   Note: 2 is the only even prime

PRODUCT OF PRIME FACTORS:

factors of 12:   1 , 2 , 3 , 4 , 6 , 12
prime factors of 12:   2 , 3
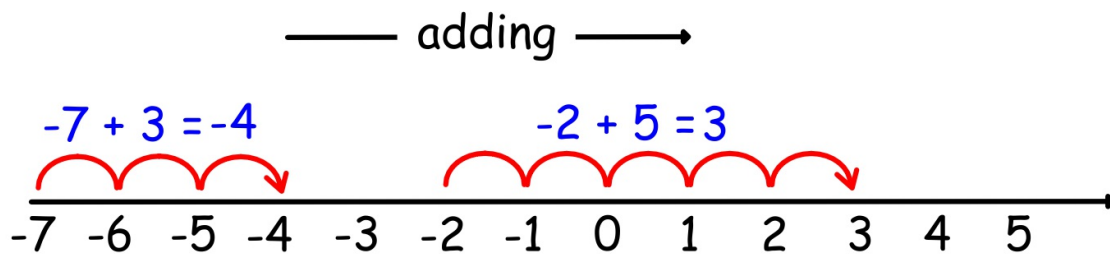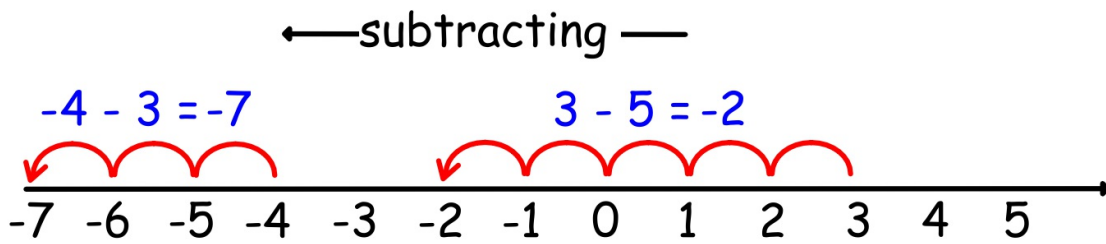product of prime factors: $12 = 2 \times 2 \times 3$

divide successively by primes
to reduce the number to 1.

2	240
2	120
2	60
2	30
3	15
5	5
	1

240 = $2 \times 2 \times 2 \times 2 \times 3 \times 5$

   = $2^4 \times 3 \times 5$ INDEX FORM

DIRECTED NUMBERS

Add and Subtract

←——subtracting ——

-4 - 3 = -7

3 - 5 = -2

-7 -6 -5 -4 -3 -2 -1   0   1   2   3   4   5

——— adding ——→

-7 + 3 = -4

-2 + 5 = 3

-7 -6 -5 -4 -3 -2 -1   0   1   2   3   4   5

$$a + (-b) = a - b$$

$$a - (-b) = a + b$$

(i) 7 + (-3) = 7 - 3 = 4

(iii) 6 - (- 2) = 6 + 2 = 8

(ii) -5 + (-1) = -5 - 1 = -6

(iv) -3 - (- 5) = -3 + 5 = 2

Multiply and Divide

P x or ÷ P = P
N x or ÷ N = P

-3 x (-6) = 18   -18 ÷ (-6) = 3

P x or ÷ N = N

3 x (-6) = -18   18 ÷ (-6) = -3

N x or ÷ P = N

-3 x 6 = -18   -18 ÷ 6 = -3

ORDER OF CALCULATION

x and ÷ before + or - change the order with brackets: BRACKETS FIRST !

(i) $-6 + 4 \times (-3)$

  $= -6 + (-12)$

  $= -18$

(ii) $(-6 + 4) \times (-3)$

  $= -2 \times (-3)$

  $= 6$

EVALUATE EXPRESSIONS
Replace the letter by its value.

$a = -2$, $b = 3$, $c = -5$

(1) $5ab$

  $= 5 \times a \times b$

  $= 5 \times (-2) \times 3$

  $= -30$

(2) $b^2 + ac$

  $= b \times b + a \times c$

  $= 3 \times 3 + (-2) \times (-5)$

  $= 9 + 10$

  $= 19$

(3) $\dfrac{c - b}{c + b}$

  $= \dfrac{-5 - 3}{-5 + 3}$

  $= \dfrac{-8}{-2}$

  $= 4$

(4) $a(b + c)$

  $= a \times (b + c)$

  $= -2 \times (3 + (-5))$

  $= -2 \times (-2)$

  $= 4$

(5) $2c^2$

  $= 2 \times c \times c$

  $= 2 \times (-5) \times (-5)$

  $= 2 \times 25$

  $= 50$

(6) $(2c)^2$

  $= (2 \times c)^2$

  $= (2 \times (-5))^2$

  $= (-10)^2$

  $= 100$

CHAPTER 2: DECIMALS

READING SCALES

0.3   2.3   3.7

```
├─────────┬─────────┬─────────┬─────────┤
0         1         2         3         4
```

0.03   0.23   0.37

```
├─────────┬─────────┬─────────┬─────────┤
0        0.1       0.2       0.3       0.4
```
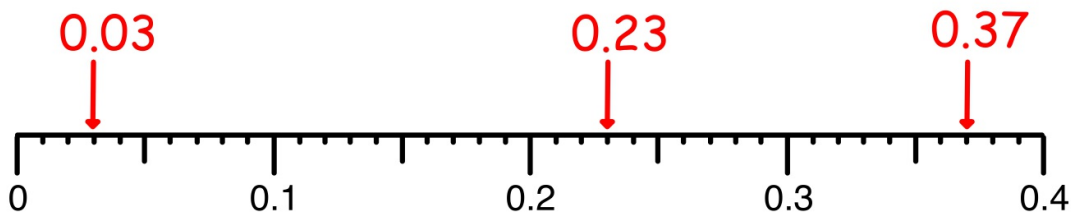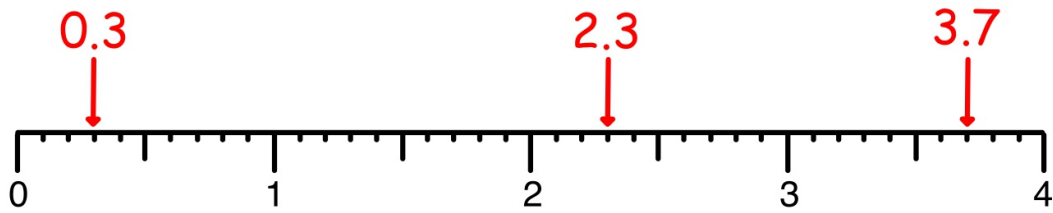
DECIMAL PLACES
the number of figures after the decimal point.

ROUNDING

(i)   12.649   (3DP) ⟨ 12.7
                       ------------ 12.65
      = 12.6   (1DP)   12.6

(ii)   3.6851   (4DP) ⟨ 3.69
                       ------------ 3.685
      = 3.69   (2DP)   3.68

SIGNIFICANT FIGURES
Indicate the accuracy of a measurement.

eg. 3400 cm = 34 m = 0.034 km

Same measurement, same accuracy: each 2 significant figures

Count the number of figures used but not zeros:
at the end of a whole number,
at the start of a decimal.

These zeros simply keep the place value of the digits.

Examples:

0.030	2 sig fig		30100	3 sig fig
5.030	4 sig fig		30100.0	6 sig fig

ROUNDING:
Round to the place value of the number of significant figures required

Examples: rounding to 2 significant figures.

(i) $\overset{\text{TH H T U}\ \ t}{5\ 7\ 5\ 1\ .\ 4}$ = 5 8 0 0   nearest HUNDRED

(ii) $\overset{\text{U}\ \ t\ \ h\ \text{th}}{0.0\ 5\ 7\ 5\ 1\ 4}$ = 0.0 5 8 nearest THOUSANDTH

NOTE: 0.058000 is wrong

ESTIMATING:

Round data to one significant figure.

Example: estimate the area of a rectangle
              43.65 cm by 18.37cm.

$$A = lb$$
$$\approx 40 \text{ cm} \times 20 \text{ cm}$$
$$\approx 800 \text{ cm}^2$$

CALCULATOR:

Read and decide the operation(s) required + , - , x , ÷
Write the calculation(s).

(1) Carpet costs £24.52 per m^2. Calculate the cost
    of 18.5 m^2.   £24.52 x 18.5 = £453.62

(2) 4.8 litres of paint covers 58.8 m^2.
    How many m^2 per litre ?   58.8 ÷ 4.8 = 12.25 m^2

(3) On a diet John drops from 105.4 kg to 96.8 kg.
     How much weight did he loose ?
                        105.4 - 96.8 = 8.6 kg

HIRE PURCHASE

HP cost = DEPOSIT + INSTALMENTS

CASH PRICE   £749.95
or
£ 99.95 deposit + 12 monthly instalments of £59.99

HP COST: 12 x £59.99 = £719.88
            £99.95 + £719.88 = £819.83

Extra cost by HP: £819.83 - £749.95
            = £69.88

FOREIGN EXCHANGE

EXCHANGE RATE: amount of foreign currency for £1.

£1 = $1.48

Change £30 to $ $30 x 1.48 = $44.40

Change $250 to £   £250 ÷ 1.48
                        = £168.91891...
                        = £168.92

ADD and SUBTRACT
remember place value

(i) 8.37 + 9.6 + 5.49

```
  T U  t h
    8 . 3 7
    9 . 6 0
  + 5 . 4 9
  ─────────
  2 3 . 4 6
    2   1   1
```

(ii) 9.48 - 5.764

```
    U  t h th
  ⁸9̶ . ¹4 ⁷8̶ ¹0
  - 5 . 7 6 4
  ───────────
    3 . 7 1 6
```

MULTIPLY and DIVIDE

(i) 4.547 × 6

```
    4 . 5 4 7
      × 6
  ─────────────
  2 7 . 2 8 2
      3   2   4
```

(ii) 27.282 ÷ 6

```
        0 4 . 5 4 7
  6 │ 2 ²7 . ³2 ²8 ⁴2
```

multiply by 10s

(i) 3.42   x 10   = 34.2

(ii) 5.87   x 100   = 587

(iii) 0.062 x 100   = 6.2

divide by 10s

(i) 34.2   ÷ 10   = 3.42

(ii) 587.   ÷ 100   = 5.87

(iii) 006.2 ÷ 100   = 0.062

(1) 3.12   ÷ 0.003

   = 3 120 ÷ 3

   = 1 040

remove decimal point from divisor: multiply both numbers by 1000

(2)   3.12 ÷ 3000

   =   1.04 ÷ 1000

   = 0.00104

divide by 3 then by 1000

(3) 3.12   x 3000

   = 3 120 x 3

   = 9 360

multiply by 1000 then by 3

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STANDARD FORM (SCIENTIFIC NOTATION)

Used to write very large and very small numbers.

Form $a \times 10^n$

$1 \leq a < 10$ ie. between 1 and 10, excluding 10

n is an INTEGER   ie. ...-3,-2,-1,0,1,2,3...

Place the decimal point after the first non-zero digit.
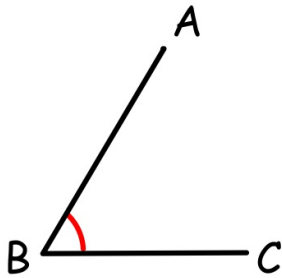Count the number of places the decimal point moves.

257000 = 2.57×10^5 2 5 7 0 0 0·

negative power - small number between 0 and 1,
                the point moves to the right.
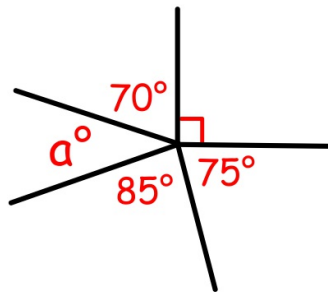
0.0000257 = 2.57×10^{-5} 0·0 0 0 0 2 5 7

CHAPTER 3: ANGLES

NAMING ANGLES: 3 capital letters and an angle sign
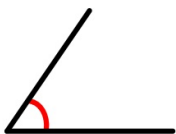the middle letter is the VERTEX.

arms: AB and BC
vertex: B
angle: ∠ABC

COMPLETE TURN:
360° around a point

70°
a°
85° 75°

```
  70
  85
  75
+ 90
─────
 320
```

```
  360
- 320
─────
   40
```

a = 40

DESCRIPTIONS (TYPES):
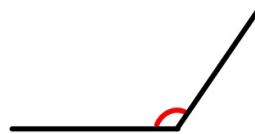
acute angle: right angle: obtuse angle: straight angle:
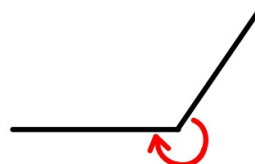between 0° and 90°   90°   between 90° and 180°   180°

Reflex angle: between 180° and 360°

130° 50°

supplementary

60°
30°
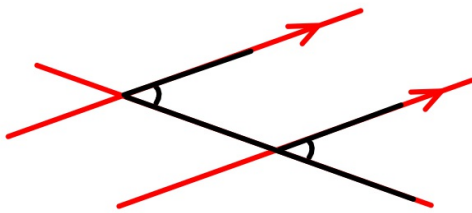
complementary

50° 50°
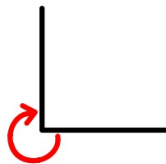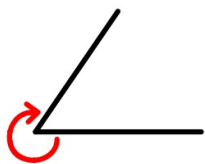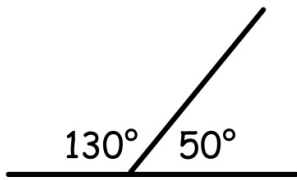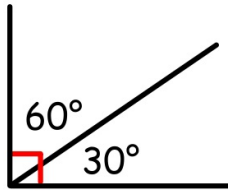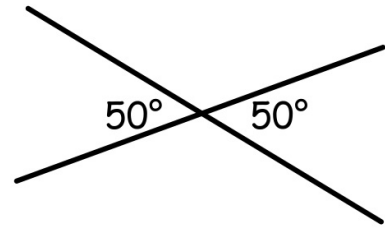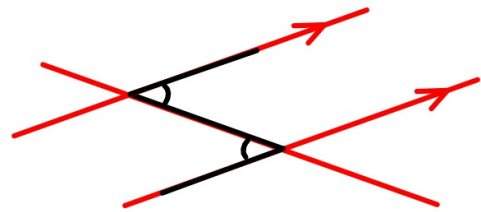
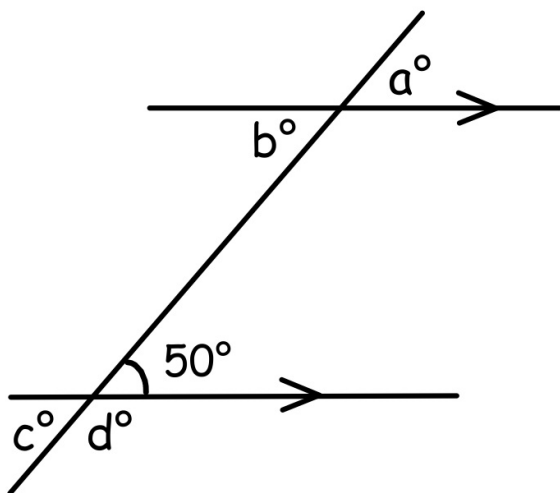vertically opposite
(X shape)

PARALLEL LINES:

corresponding angles equal
(F shape)

alternate angles equal
(Z shape)

a°
b°
50°
c° d°

a = 50   corresponding (F shape)

b = 50   alternate (Z shape)

c = 50   vertically opposite
             (X shape)

d = 130 supplementary   1 8 0
                              - 5 0
                              1 3 0

CHAPTER 4: ALGEBRA

SHORT FORMS

multiplication is repeated addition

(i) $7 + 7 + 7 + 7 + 7 + 7$ $= 6 \times 7$

(ii) $a + a + a + a + a + a$ $= 6 \times a$ $= 6a$

(iii) $a^2 + a^2 + a^2 + a^2 + a^2 + a^2$ $= 6 \times a^2$ $= 6a^2$

a power is repeated multiplication

(i) $7 \times 7 \times 7 \times 7 \times 7 \times 7$ $= 7^6$

(ii) $a \times a \times a \times a \times a \times a$ $= a^6$

"Like Terms" can be added/subtracted.
(same letter combinations)

(1) $5a - 2b + 3a$ (2) $5ab - 2ba + ab$ (3) $7a^2 - 4a - 6a^2$
    $= 5a + 3a - 2b$ $= 5ab - 2ab + 1ab$ $= 7a^2 - 6a^2 - 4a$
    $= 8a - 2b$ $= 4ab$ $= a^2 - 4a$

Multiplication combines letters and numbers.

(4) $2a^2 \times 4ab$ (5) $3c^2 \times 2c - 5 \times c \times c \times d$
$= 2 \times a \times a \times 4 \times a \times b$ $=$ $6c^3 - 5c^2d$
$= 2 \times 4 \times a \times a \times a \times b$
$= 8a^3b$

BRACKET BREAKING
$$a \times (b + c) = a \times b + a \times c$$

multiply each term in the brackets

(1) 6 (2w - 3y)

   = 12w - 18y

(2) 8p (2p + r)

   = $16p^2$ + 8pr

remove brackets then simplify:

(3) 2 (3p + 5) - 7

   = 6p + 10 - 7

   = 6p + 3

(4) n (4n - 3) + n^2

   = $4n^2$ - 3n + n^2

   = $5n^2$ - 3n

FACTORISATION
$$a \times b + a \times c = a \times (b + c)$$

To FULLY factorise a is the Highest Common Factor

(1) 12w - 18y

  = 6 (2w - 3y)

(2) $4n^2$ + 5mn - n

  = n (4n + 5m - 1)

(3) $16p^2$ + 8pr

  = 8p (2p + r)

(4) 1.4 × 3.6 - 1.4 × 1.6

  = 1.4 × (3.6 - 1.6)

  = 1.4 × 2

  = 2.8

AVERAGES:

mean = $\dfrac{\text{total of all results}}{\text{number of results}}$

median: the middle result of the ordered results
(if an even number of results - find the mean of the middle two results)

mode: the most common result

SPREAD:

range = highest result - lowest result

19,14,15,19,16,22,16,20,24,14,25,18,27,12,16

mean = $^{277}/_{15}$ = 18.466... = 18.5

15 ordered results: 8th result in the middle
12,14,14,15,16,16,16,18,19,19,20,22,24,25,27

median = 18
  mode = 16
  range = 27 - 12 = 15

median for even number of results:

16 ordered results: 8th/9th results in the middle
12,14,14,15,16,16,16,18,19,19,20,22,24,25,27,30

median = (18+19) ÷ 2 = 18.5

MEAN PROBLEMS   Total = mean x number of results

(1) 15 girls, mean height 1.2m   1.2 m x 15   = 18 m
     10 boys, mean height 1.4m   1.4 m x 10   = 14 m

     Find the mean height of   total height   32 m
     the group of 25 children ?   mean = 32 ÷ 25 = 1.28 m

(2) 4 boys have mean weight 60 kg   60 Kg x 4 = 240 Kg
     When John is included the   62 Kg x 5 = 310 Kg
     mean weight is 62 kg
                                        310 - 240 = 70 Kg
     Find the weight of John.

FREQUENCY TABLES

If there is a lot of data the information is easier to present in a frequency distribution table.

TEST SCORES: 1,1,1,2,2,2,2,2,3,3,3,3,3,3,3,4,4,4,5,5

result	tally	frequency
1	III	3
2	IIII	5
3	IIII II	7
4	III	3
5	II	2

MEAN FROM A FREQUENCY TABLE:

A result x frequency column is included.

$$\text{mean} = \frac{\text{total of: result} \times \text{frequency}}{\text{total frequency}}$$

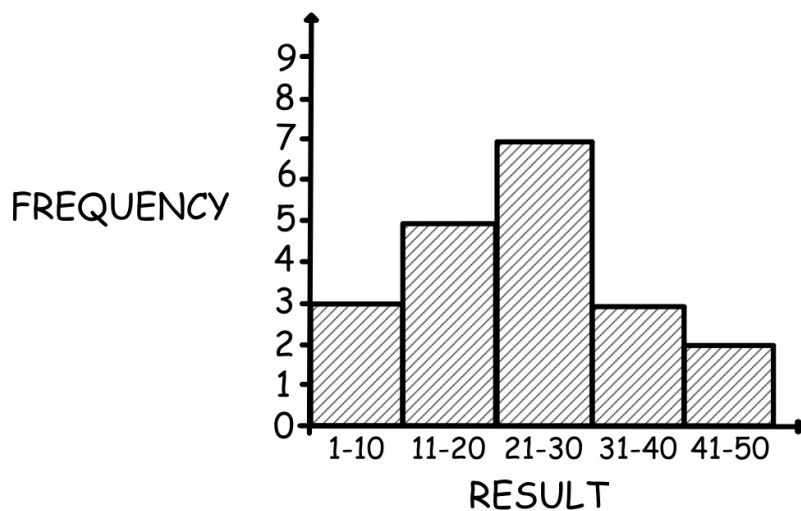| result	frequency	result x frequency	
1	3	3	
2	5	10	
3	7	21	mean = 56 ÷ 20
4	3	12	= 2.8
5	2	10	
TOTALS	20	56	

GROUPED FREQUENCY TABLES
Data can be grouped into class intervals

TEST SCORES: 3,5,8 , 11,13,17,17,19,
22,23,24,25,26,28,29,
33,38,39 , 44,47

result	tally	frequency
1 - 10	III	3
11 - 20	IIII	5
21 - 30	IIII II	7
31 - 40	III	3
41 - 50	II	2

The results can be shown on a frequency diagram.

The MODAL CLASS has the highest frequency.

MODAL CLASS is 21-30