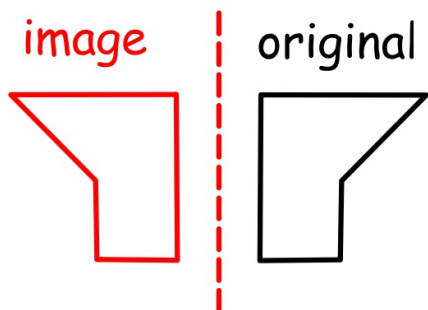
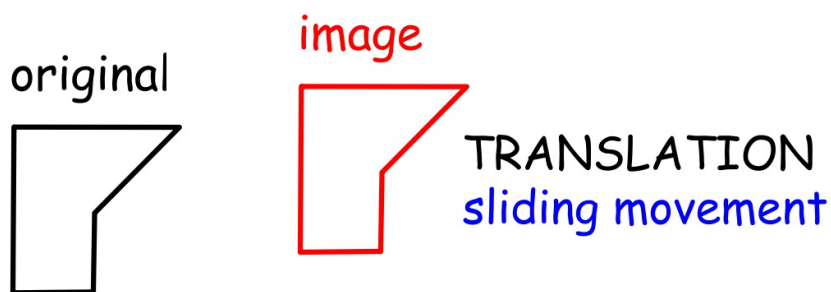
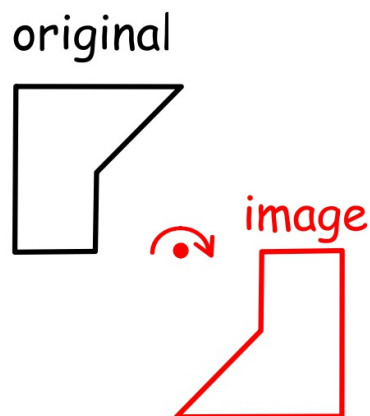


CHAPTER 10: TRANSFORMATIONS

REFLECTION
folding over a line



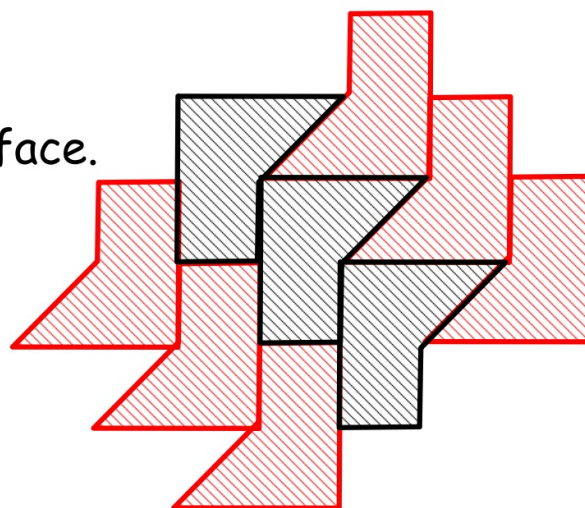
ROTATION
turning about a point



TILINGS

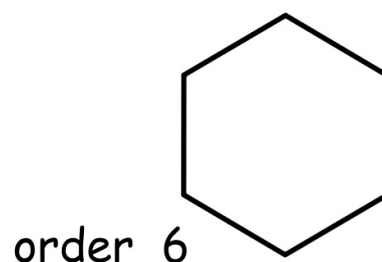
CONGRUENT tiles cover a surface.
(identical)

No gaps or overlap.
Can extend in any direction.



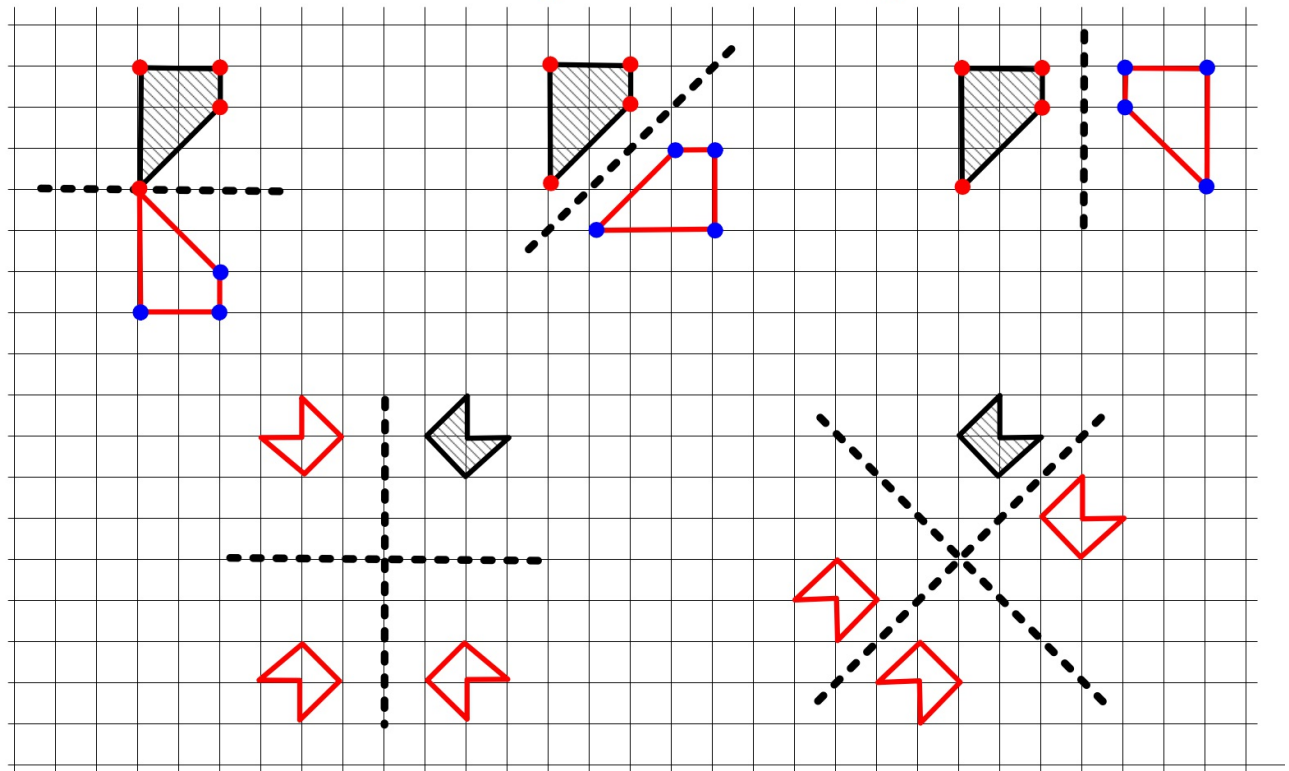
ORDER OF ROTATIONAL SYMMETRY

The number of times a shape fits itself under one turn.

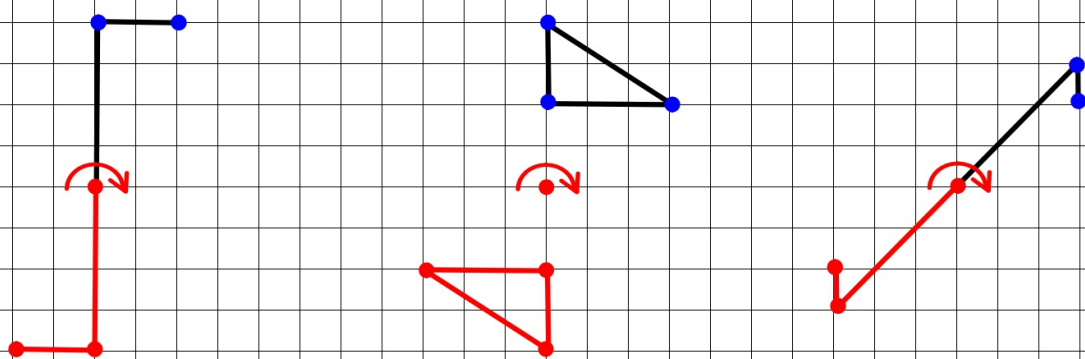


Line symmetry

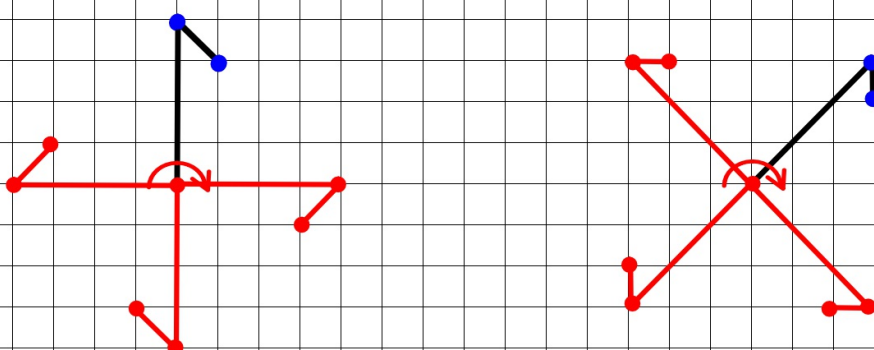
Reflect the corners and join for the image.



Half-turn symmetry

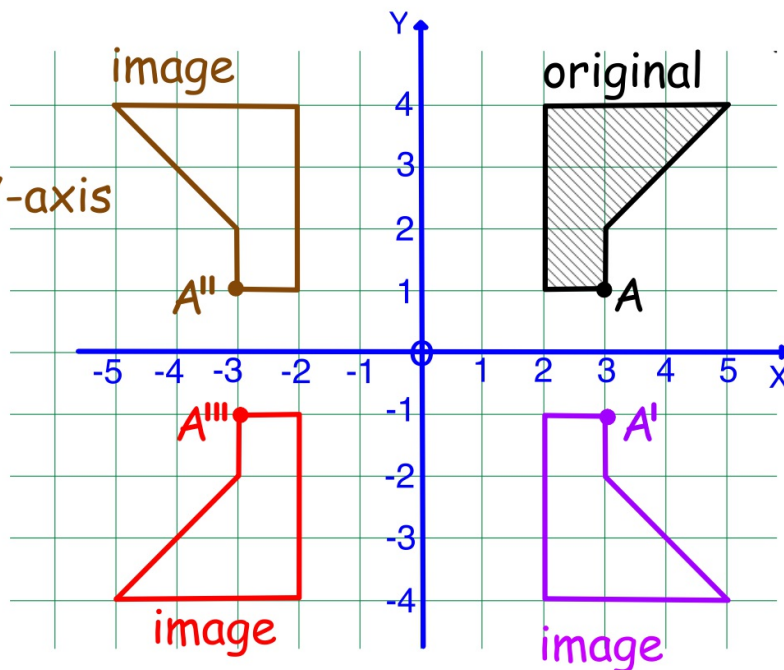


Quarter-turn symmetry



COORDINATES

reflection in the Y-axis

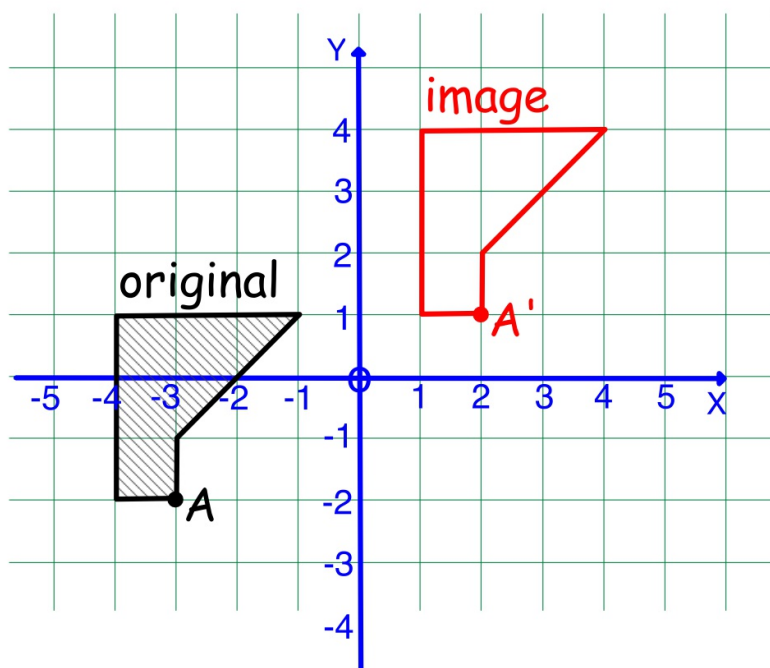


half-turn about O

reflection in the X-axis

image under $A(-3,-2) \rightarrow A'(2,1)$

all points move
5 right , 3 up



ENLARGEMENT and REDUCTION

Angles are unchanged.

Sides are enlarged/reduced by a SCALE FACTOR.

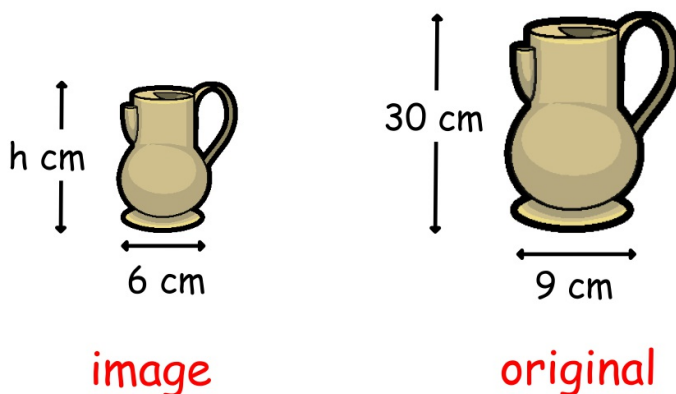
ENLARGEMENT: $SF > 1$

REDUCTION: $0 < SF < 1$

The shape with the dimension to be found is the image.

$$\text{Scale Factor} = \frac{\text{image size}}{\text{original size}}$$

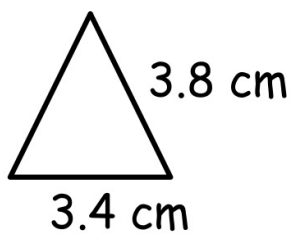
One jug is a reduction of the other.



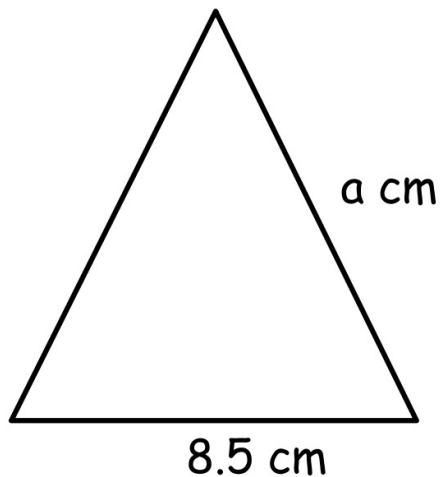
$$SF = \frac{6}{9} = \frac{2}{3}$$

$$\begin{aligned} h &= 30 \times \frac{2}{3} \\ &= 30 \div 3 \times 2 \\ &= 20 \end{aligned}$$

ENLARGEMENT



original

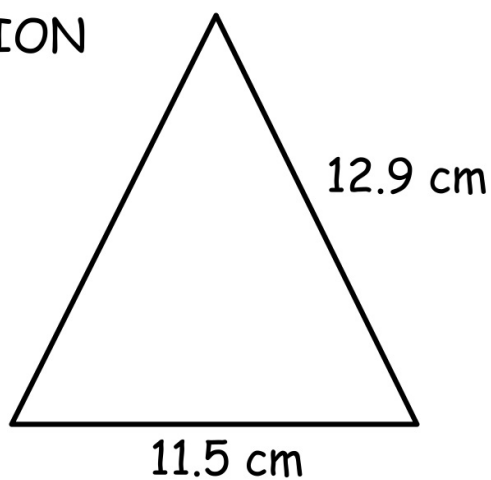


image

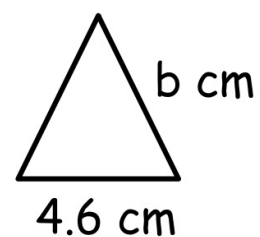
$$SF = \frac{8.5}{3.4} = 2.5$$

$$a = 3.8 \times 2.5 = 9.5$$

REDUCTION



original



image

$$SF = \frac{4.6}{11.5} = 0.4$$

$$b = 12.9 \times 0.4 = 5.16$$

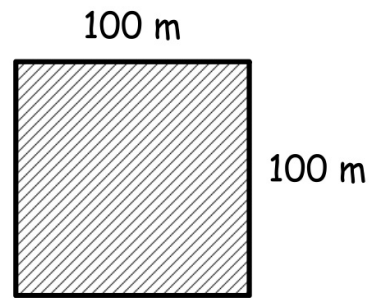
CHAPTER 11: AREA

AREA UNITS

$$1 \text{ cm}^2 = 100 \text{ mm}^2$$

$$1 \text{ m}^2 = 10\,000 \text{ cm}^2$$

$$1 \text{ km}^2 = 1\,000\,000 \text{ m}^2$$



$$10\,000 \text{ m}^2 = 1 \text{ hectare}$$

$$250 \text{ mm}^2 = 2.5 \div 100 = 2.5 \text{ cm}^2$$

$$0.4 \text{ m}^2 = 0.4 \times 10\,000 = 4000 \text{ cm}^2$$

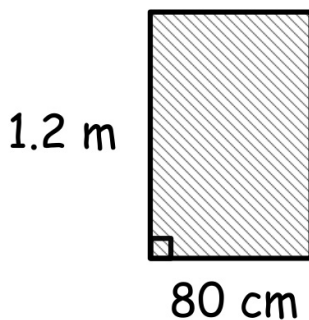
$$0.02 \text{ km}^2 = 0.02 \times 1\,000\,000 = 20\,000 \text{ m}^2$$

$$35\,000 \text{ m}^2 = 35\,000 \div 10\,000 = 3.5 \text{ hectares}$$

RECTANGLE

$$A = lb$$

NOTE: match length units to required area units.
eg. cm for cm^2



$$\begin{aligned} A &= lb \\ &= 1.2 \times 0.8 \\ &= 0.96 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A &= lb \\ &= 120 \times 80 \\ &= 9600 \text{ cm}^2 \end{aligned}$$

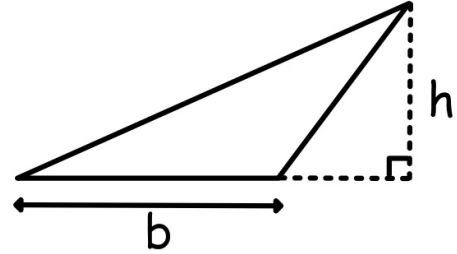
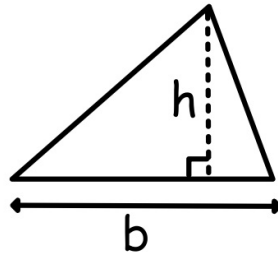
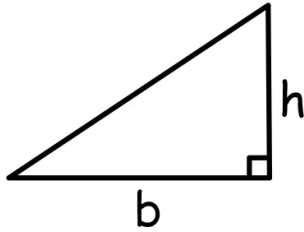
TRIANGLES

$$A = \frac{1}{2} bh$$

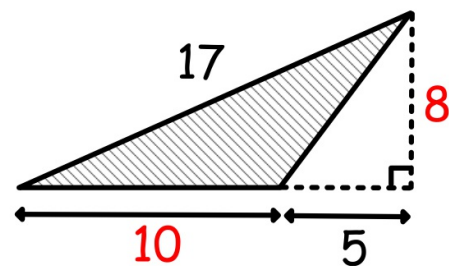
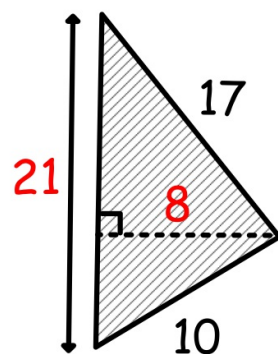
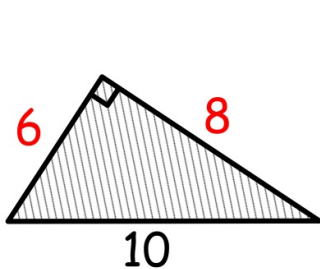
The height and the base are at 90° .

(altitude)

(perpendicular)



NOTE: only base and height required;
ignore extra dimensions (sizes).



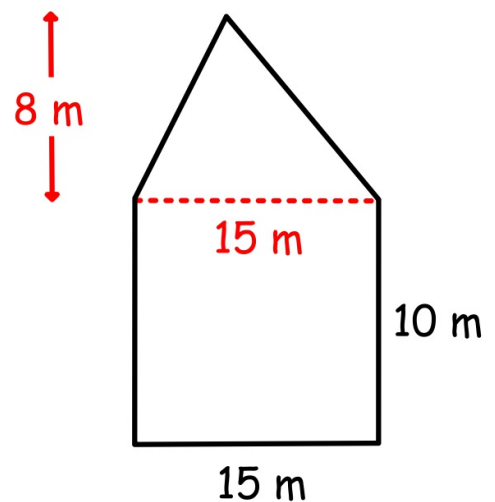
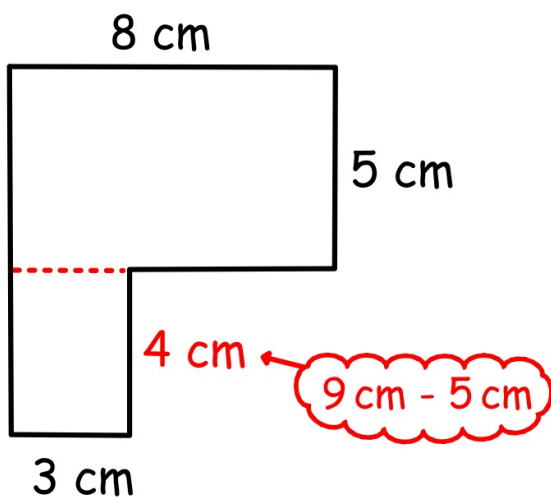
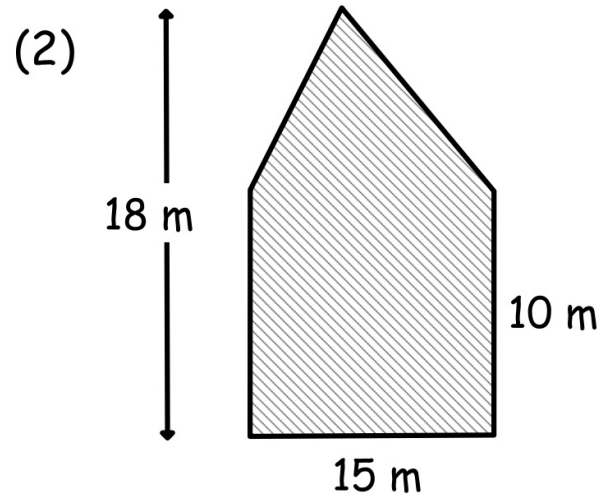
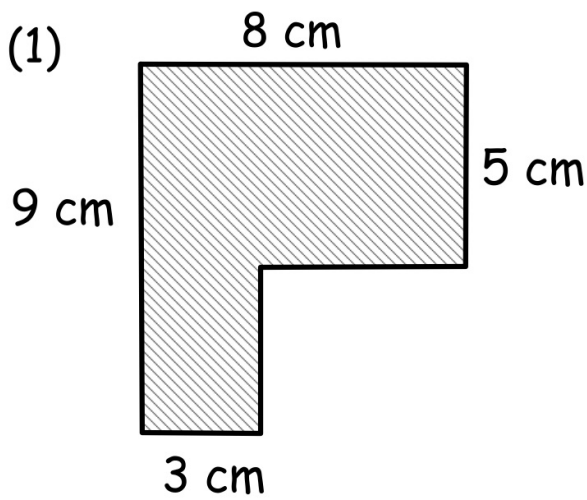
$$\begin{aligned} A &= \frac{1}{2} bh \\ &= 6 \times 8 \div 2 \\ &= 24 \text{ units}^2 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2} bh \\ &= 21 \times 8 \div 2 \\ &= 84 \text{ units}^2 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2} bh \\ &= 10 \times 8 \div 2 \\ &= 40 \text{ units}^2 \end{aligned}$$

COMPOSITE SHAPES

Formed from rectangles and triangles



$$\begin{aligned}
 A &= lb & A &= lb \\
 &= 8 \times 5 & &= 4 \times 3 \\
 &= 40 \text{ cm}^2 & &= 12 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 &40 \text{ cm}^2 + 12 \text{ cm}^2 \\
 &= 52 \text{ cm}^2
 \end{aligned}$$

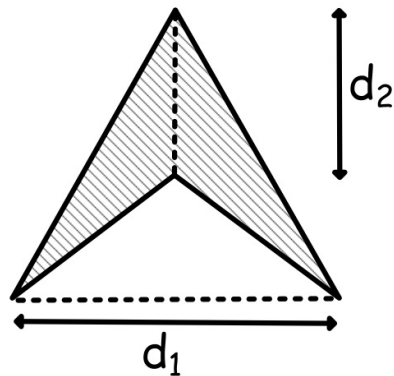
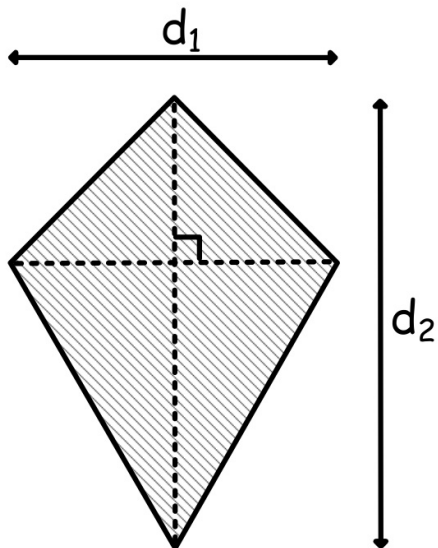
$$\begin{aligned}
 A &= lb & A &= \frac{1}{2} bh \\
 &= 15 \times 10 & &= 15 \times 8 \div 2 \\
 &= 150 \text{ m}^2 & &= 60 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 &150 \text{ m}^2 + 60 \text{ m}^2 \\
 &= 210 \text{ m}^2
 \end{aligned}$$

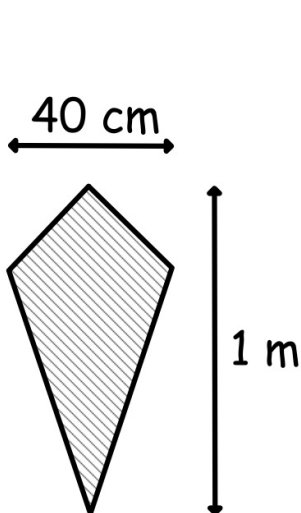
KITE and RHOMBUS

$A = \frac{1}{2}$ the product of the diagonals

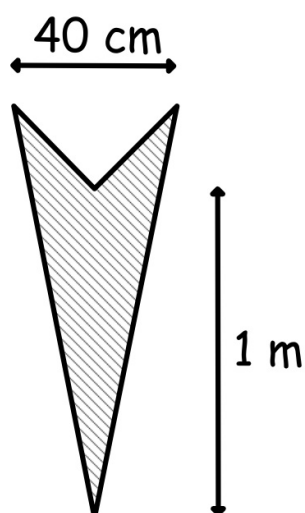
$$A = \frac{1}{2} d_1 d_2$$



Ensure the units match.



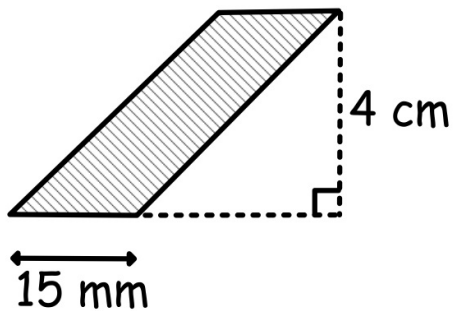
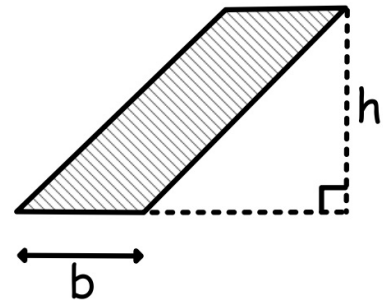
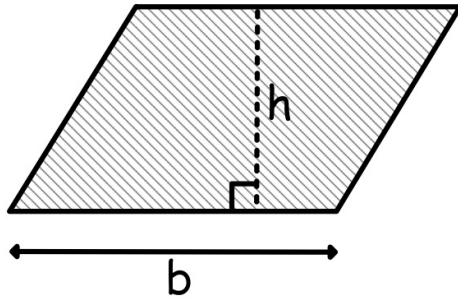
$$\begin{aligned} A &= \frac{1}{2} d_1 d_2 \\ &= 40 \times 100 \div 2 \\ &= \underline{\underline{2000\text{ cm}^2}} \end{aligned}$$



$$\begin{aligned} \text{or } A &= \frac{1}{2} d_1 d_2 \\ &= 0.4 \times 1 \div 2 \\ &= \underline{\underline{0.2\text{ m}^2}} \end{aligned}$$

PARALLELOGRAM

$$A = bh$$

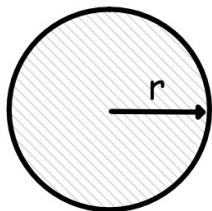


$$\begin{aligned} A &= bh \\ &= 1.5 \times 4 \\ &= \underline{\underline{6 \text{ cm}^2}} \end{aligned}$$

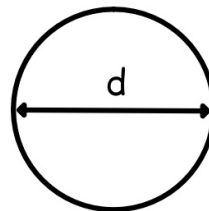
$$\begin{aligned} \text{or } A &= bh \\ &= 15 \times 40 \\ &= \underline{\underline{600 \text{ mm}^2}} \end{aligned}$$

CIRCLES

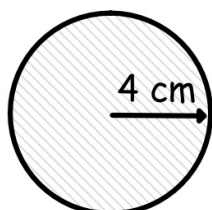
AREA: $A = \pi r^2$



CIRCUMFERENCE: $C = \pi d$



Remember $r = \frac{1}{2} d$ and $\pi \approx 3.14$

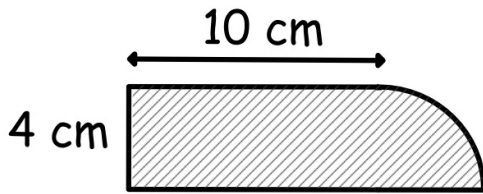


$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times 4 \times 4 \\ &= 50.24 \text{ cm}^2 \end{aligned}$$

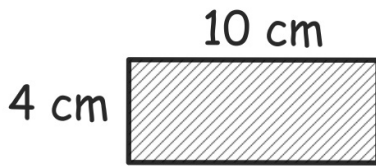
$$\begin{aligned} C &= \pi d \\ &= 3.14 \times 8 \\ &= 25.12 \text{ cm} \end{aligned}$$

COMPOSITE SHAPES

Identify the rectangle and circle parts.



AREA:



$$\begin{aligned} A &= lb \\ &= 4 \times 10 \\ &= 40 \text{ cm}^2 \end{aligned}$$

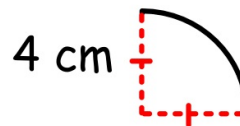
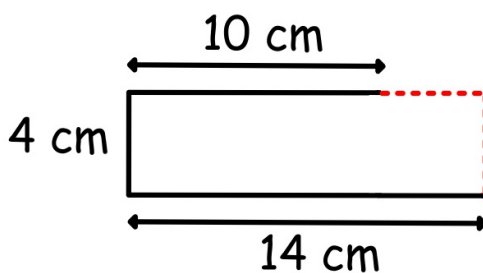


$$\begin{aligned} A &= \pi r^2 \\ &= 3.14 \times 4 \times 4 \\ &= 50.24 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A &= 50.24 \div 4 \\ &= 12.56 \text{ cm}^2 \end{aligned}$$

$$\text{Total Area} = 12.56 + 40 = \underline{\underline{52.56 \text{ cm}^2}}$$

PERIMETER:



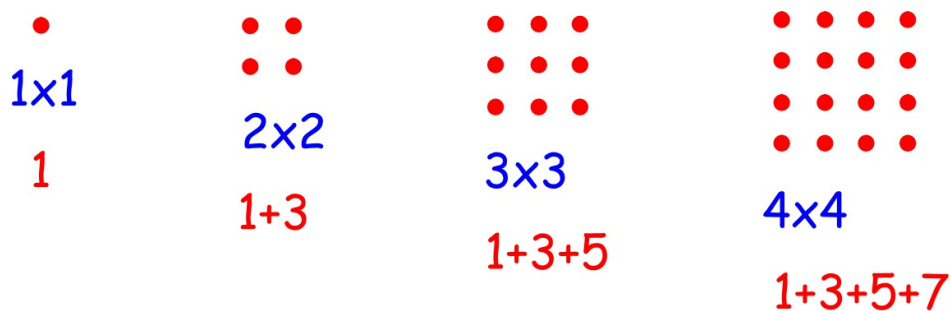
$$\begin{aligned} C &= \pi d \\ &= 3.14 \times 8 \\ &= 25.12 \text{ cm} \end{aligned}$$

$$\begin{aligned} &25.12 \div 4 \\ &= 6.28 \text{ cm} \end{aligned}$$

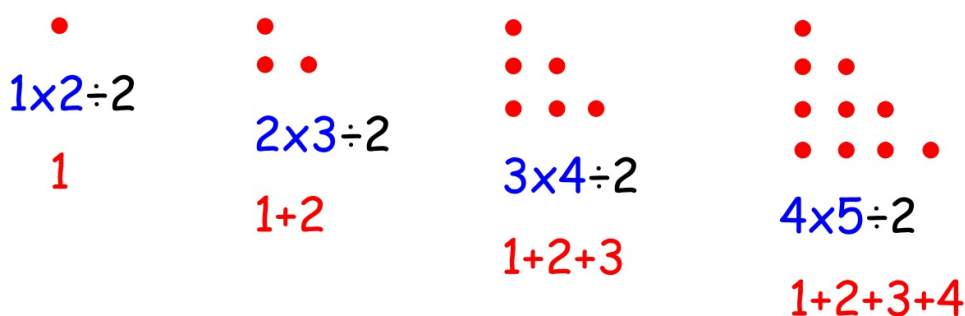
$$\text{Perimeter} = 4 + 10 + 14 + 6.28 = \underline{\underline{34.28 \text{ cm}}}$$

CHAPTER 12: SEQUENCES

square numbers 0, 1, 4, 9, 16, 25 ...



triangular numbers 0, 1, 3, 6, 10, 15 ...



GENERALISE



- find (a) a **formula** for the number of matches.
 (b) the number of matches for **10 triangles**.
 (c) the number of triangles for **51 matches**.

(a)	triangles	1	2	3	4	5	T
	matches	3	5	7	9	11	$2T + 1$



Formula: $M = 2T + 1$

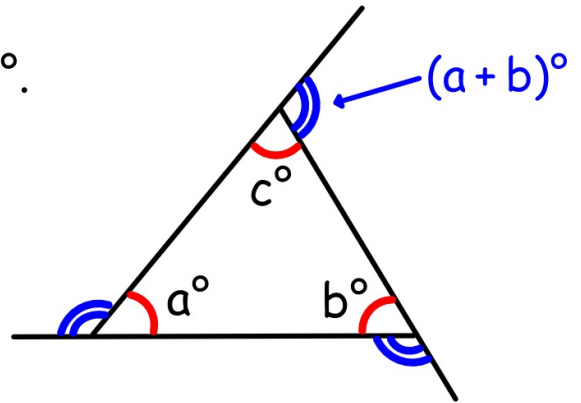
(b) $M = 2 \times 10 + 1 = 21$

(c) $2T + 1 = 51$
 $2T = 50$
 $T = 25$

CHAPTER 13: TRIANGLES

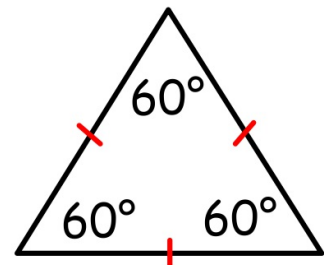
INTERIOR angles add up to 180° .

-  interior angles
-  exterior angles



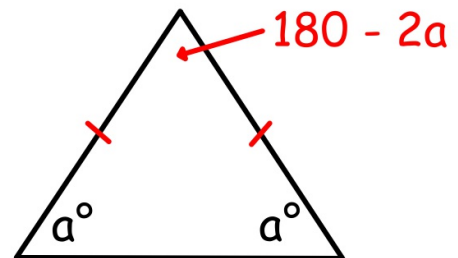
EQUILATERAL TRIANGLES

Three equal sides,
three equal angles of 60° .

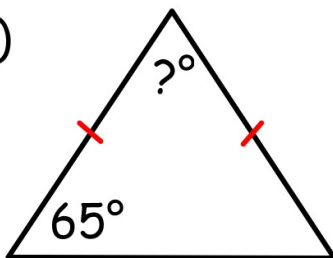


ISOSCELES TRIANGLES

Two equal sides, two equal angles.

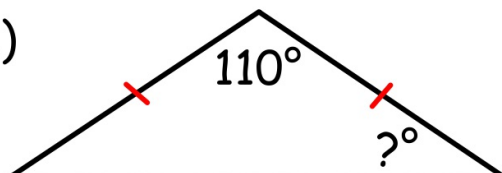


(1)



$$\begin{array}{r} 65 \\ + 65 \\ \hline 130 \end{array} \qquad \begin{array}{r} 180 \\ - 130 \\ \hline 50 \end{array}$$

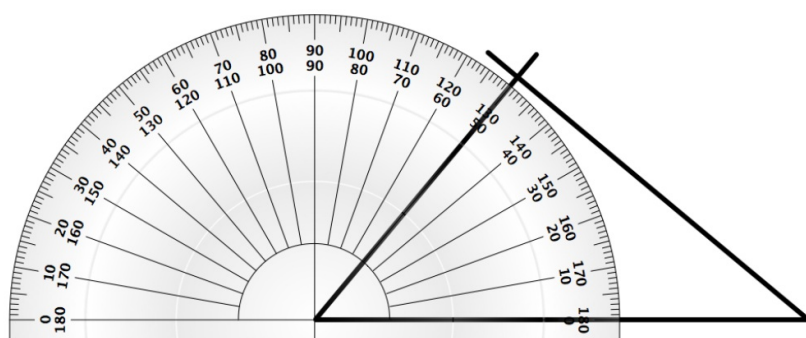
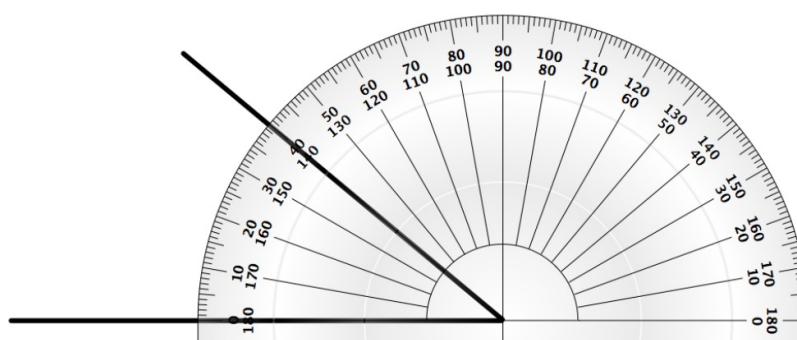
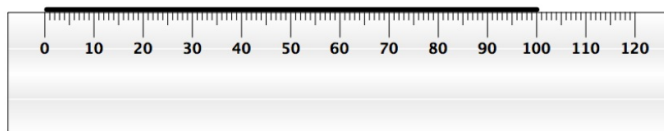
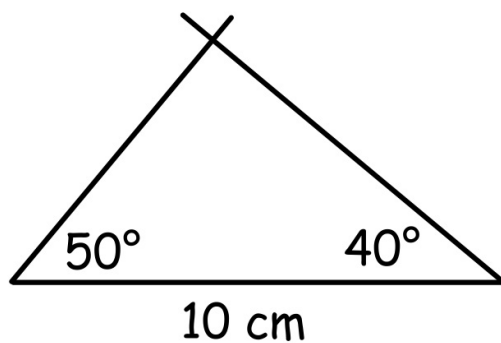
(2)



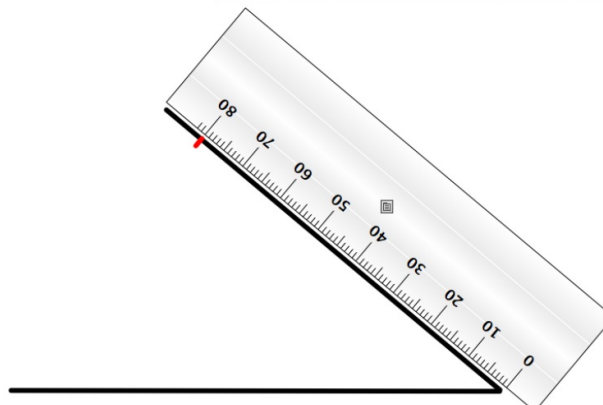
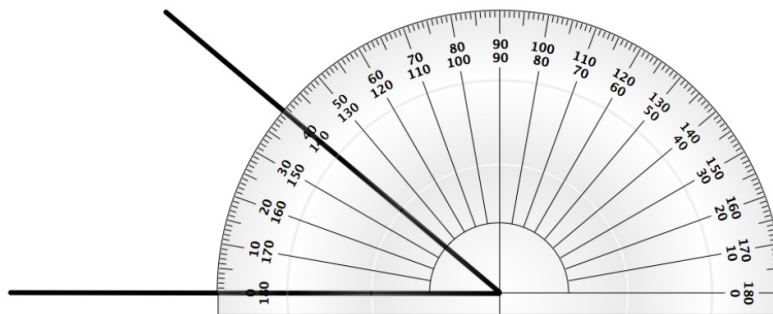
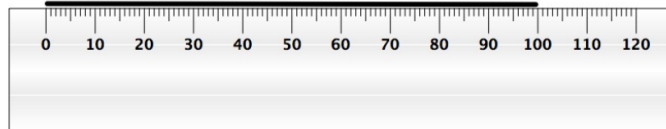
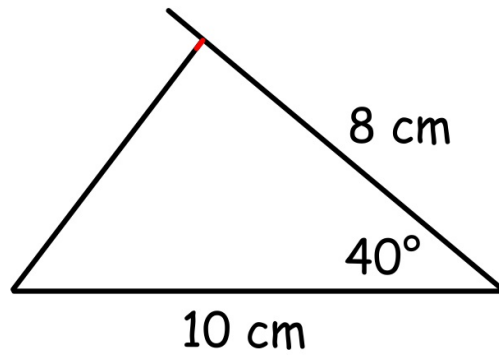
$$\begin{array}{r} 180 \\ - 110 \\ \hline 70 \end{array} \qquad \begin{array}{r} 35 \\ 2 \overline{) 70} \end{array}$$

CONSTRUCTING TRIANGLES

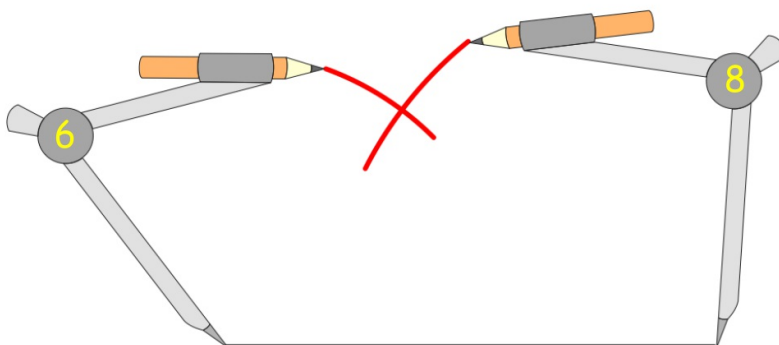
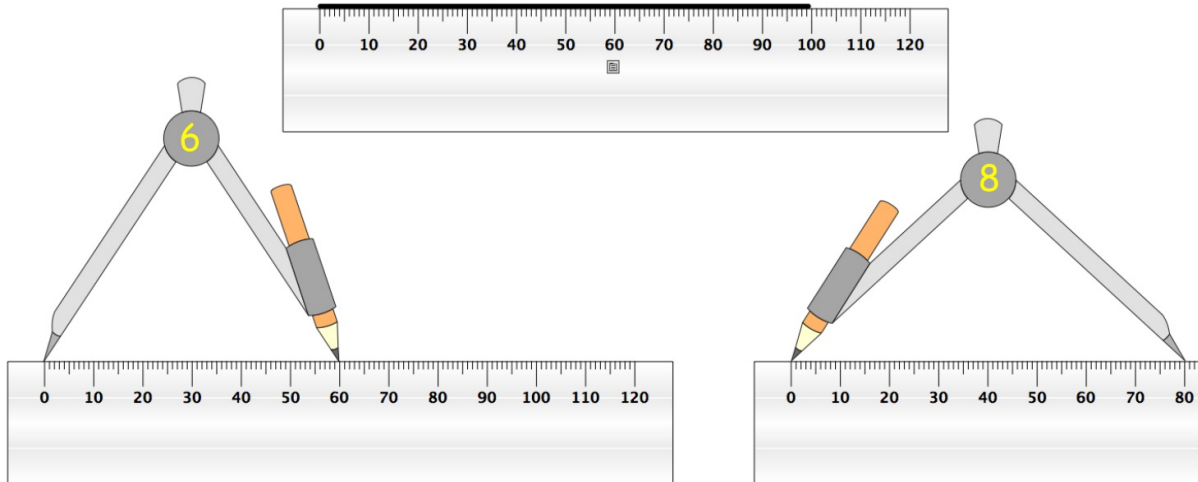
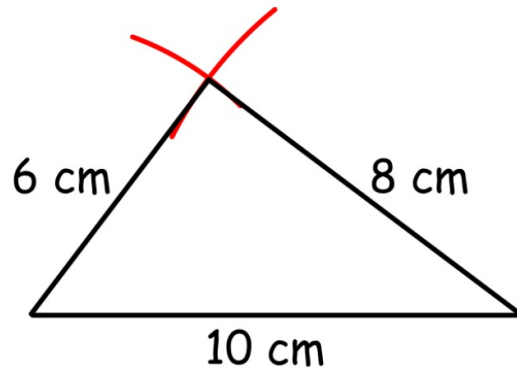
Given 1 side and 2 angles: protractor construction



Given 2 sides and angle between: protractor construction



Given all 3 sides: compass construction



CHAPTER 14: RATIO and PROPORTION

RATIO

Comparing the sizes of quantities, so there are no units.

school trip: ratio of teachers to pupils is 1:10

model ship: scale 1:400

TV screen: width to height 4:3 (aspect ratio)

Fully simplify ratios:

$$\begin{aligned} & a : b \\ & = a \div \text{HCF} : b \div \text{HCF} \end{aligned}$$

$$\begin{aligned} (1) \quad & 8 : 12 \quad \text{divide by 4} \\ & = 2 : 3 \end{aligned}$$

$$\begin{aligned} (2) \quad & 6 \text{ kg} : 900 \text{ g} \quad \text{same units} \\ & = 6000 \text{ g} : 900 \text{ g} \quad \text{divide by 100} \\ & = 60 : 9 \quad \text{divide by 3} \\ & = 20 : 3 \end{aligned}$$

UNITARY RATIO

Express 5 : 4 in the form (a) 1 : n (b) n : 1

$$\begin{aligned} (a) \quad & 5 : 4 \quad \text{divide by 5} \\ & = 1 : 0.8 \end{aligned}$$

$$\begin{aligned} (b) \quad & 5 : 4 \quad \text{divide by 4} \\ & = 1.25 : 1 \end{aligned}$$

Can multiply up ratios:

$$\begin{aligned} & a : b \\ & = a \times n : b \times n \end{aligned}$$

Mix yellow and blue paint in the ratio 2 : 3
How much blue paint for 8 tins of yellow ?

$$\begin{aligned} & Y : B \quad \text{yellow tins: } 8 \div 2 = 4 \\ = & 2 : 3 \quad \text{multiply by 4} \\ = & 8 : 12 \end{aligned}$$

12 tins of blue

SHARING

Tim and Tom buy 60 chocolates.
Tim contributes £3 and Tom £2.
How many chocolates should each get ?

share 60 chocs in ratio 3:2

$$\begin{aligned} \text{number of shares} & 3 + 2 = 5 \text{ shares} \\ \text{one share} & 60 \div 5 = 12 \text{ chocs} \end{aligned}$$

$$\begin{aligned} \text{Tim } 3 \text{ shares} & 3 \times 12 = 36 \text{ chocs} \\ \text{Tom } 2 \text{ shares} & 2 \times 12 = 24 \text{ chocs} \end{aligned}$$

DIRECT PROPORTION:

Changing one quantity by some factor changes the other quantity by the same factor

2 x speed results in 2 x distance travelled
 $\frac{1}{2}$ x speed results in $\frac{1}{2}$ x distance travelled
 $\frac{2}{3}$ x speed results in $\frac{2}{3}$ x distance travelled

DISTANCE and SPEED are directly proportional

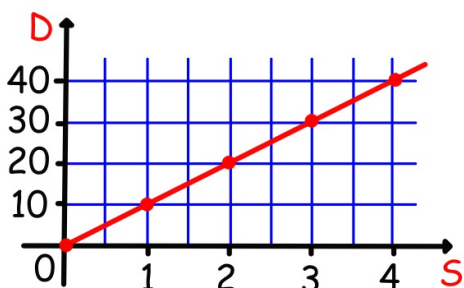
One quantity is a multiple of the other.

speed (S)	0	1	2	3	4
distance (D)	0	10	20	30	40

$$D = 10 \times S$$

GRAPHS

The graph is a straight line through the origin.



- (1) Ten books cost £36.
 (a) Find the cost of seven books.
 (b) How many books for £54 ?

Rate:
 £3.60 per book

(a) 10 books \longrightarrow £36
 1 book \longrightarrow £36 \div 10
 7 books \longrightarrow £36 \div 10 \times 7 = £25.20

(b) £36 \longrightarrow 10 books
 £1 \longrightarrow 10 \div 36 books
 £54 \longrightarrow 10 \div 36 \times 54 = 15 books

or
 £54 \div £3.60

- (2) 60 cm³ of a liquid weighs 72 g.
 (a) Find the weight of 40 cm³
 (b) Find the volume of 84 g

Rate:
 1.2 g per cm³

(a) 60 cm³ \longrightarrow 72 g
 1 cm³ \longrightarrow 72 \div 60
 40 cm³ \longrightarrow 72 \div 60 \times 40 = 48 g

(b) 72 g \longrightarrow 60 cm³
 1 g \longrightarrow 60 \div 72
 84 g \longrightarrow 60 \div 72 \times 84 = 70 cm³

or
 84 g \div 1.2 g/cm³

INVERSE PROPORTION:

Changing one quantity by some factor changes the other quantity by the **RECIPROCAL** of the factor

2 x speed results in $\frac{1}{2}$ x time taken
 $\frac{1}{2}$ x speed results in 2 x time taker
 $\frac{2}{3}$ x speed results in $\frac{3}{2}$ x time taken

TIME and **SPEED** are inversely proportional

The quantities multiply to the same product.

speed (S)	1	2	3	4	$S \times T = 12$
time (T)	12	6	4	3	

A school has money to buy 50 books at £18 each.
Price increases to £20. How many books can be bought?

50 books at £18 each total money = $50 \times £18 = £900$

£900 at £20 each N° books = $900 \div 20 = \underline{\underline{45 \text{ books}}}$

OR

18 £/book \longrightarrow 50 books

1 £/book \longrightarrow 50×18 (900 books)

20 £/book \longrightarrow $50 \times 18 \div 20 = \underline{\underline{45 \text{ books}}}$