

CHAPTER 15: FRACTIONS and PERCENTAGES

$$\begin{array}{ccccc} 1\% & 5\% & 10\% & 20\% & 25\% \\ \frac{1}{100} & \frac{1}{20} & \frac{1}{10} & \frac{1}{5} & \frac{1}{4} \end{array}$$

$$\begin{array}{cccc} 33\frac{1}{3}\% & 50\% & 66\frac{2}{3}\% & 75\% \\ \frac{1}{3} & \frac{1}{2} & \frac{2}{3} & \frac{3}{4} \end{array}$$

(1) Find 4% of 250 kg

non-calculator:

$$\begin{array}{l} 1\% \quad 250 \div 100 = 2.5 \text{ kg} \\ 4\% \quad 2.5 \times 4 = 10 \text{ kg} \end{array}$$

by calculator:

$$\begin{array}{l} 250 \div 100 \times 4 \\ = 10 \text{ kg} \end{array}$$

(2) Find $66\frac{2}{3}\%$ of 18 m

$\frac{2}{3}$ of 18 m

$$\begin{array}{l} 18 \div 3 = 6 \text{ m} \\ 6 \times 2 = 12 \text{ m} \end{array}$$

(3) Find 35% of £240

non-calculator:

$$\boxed{10\% \quad \pounds 240 \div 10 = \pounds 24}$$

$$\begin{array}{l} 30\% \quad \pounds 24 \times 3 = \pounds 72 \\ 5\% \quad \pounds 24 \div 2 = \underline{\pounds 12} \\ \pounds 84 \end{array}$$

by calculator:

$$\begin{array}{l} \pounds 240 \div 100 \times 35 \\ = \pounds 84 \end{array}$$

EXPRESSING CHANGE AS A PERCENTAGE

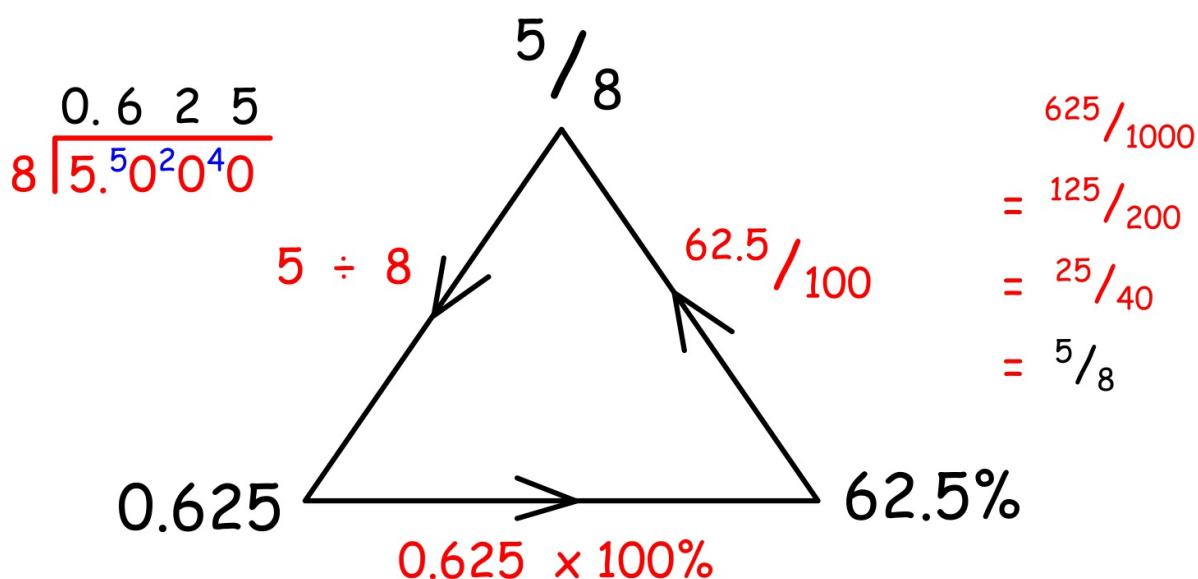
$$\% \text{ CHANGE} = \frac{\text{CHANGE}}{\text{START VALUE}} \times 100\%$$

A £15000 car is resold for £12000. Find the % loss.

$$\text{actual loss} = \text{£15000} - \text{£12000} = \text{£3000}$$

$$\begin{aligned} \% \text{ loss} &= \frac{\text{loss}}{\text{start value}} \times 100\% \\ &= \frac{\text{£3000}}{\text{£15000}} \times 100\% \\ &= 20\% \end{aligned}$$

SWITCHING BETWEEN FORMS



EQUAL FRACTIONS

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

SIMPLIFYING:

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

MIXED NUMBERS

$$\begin{aligned} 2\frac{3}{4} &= 2 + \frac{3}{4} \\ &= \frac{8}{4} + \frac{3}{4} \\ &= \frac{11}{4} \end{aligned}$$

$$2\frac{3}{4} = \frac{11}{4}$$

$4 \times 2 + 3 = 11$

$$\begin{aligned} \frac{11}{4} &= \frac{8}{4} + \frac{3}{4} \\ &= 2 + \frac{3}{4} \\ &= 2\frac{3}{4} \end{aligned}$$

$$\frac{11}{4} = 2\frac{3}{4}$$

$11 \div 4 = 2 \text{ R } 3$

ADD and SUBTRACT requires a common denominator, the LCM (least common multiple)

$$\begin{aligned} (1) \quad \frac{2}{5} + \frac{3}{10} \\ &= \frac{4}{10} + \frac{3}{10} \\ &= \frac{7}{10} \end{aligned}$$

$$\begin{aligned} (2) \quad \frac{5}{6} - \frac{2}{9} \\ &= \frac{15}{18} - \frac{4}{18} \\ &= \frac{11}{18} \end{aligned}$$

for mixed numbers treat fractions and whole numbers separately.

$$\begin{aligned} (3) \quad 12\frac{5}{6} - 3\frac{2}{9} \\ &= 12\frac{15}{18} - 3\frac{4}{18} \\ &= 9\frac{11}{18} \end{aligned}$$

MULTIPLY $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

"of" means \times

change mixed numbers to "top-heavy" fractions

(1) $\frac{3}{10}$ of $2\frac{3}{4}$
 $= \frac{3}{10} \times \frac{11}{4}$
 $= \frac{33}{40}$

(2) $1\frac{2}{3} \times 3\frac{1}{5}$
 $= \frac{5}{3} \times \frac{16}{5}$
 $= \frac{80}{15}$
 $= \frac{16}{3}$
 $= 5\frac{1}{3}$

DIVIDE $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$
 RECIPROCAL

(1) $\frac{5}{6} \div \frac{3}{7}$
 $= \frac{5}{6} \times \frac{7}{3}$
 $= \frac{35}{18}$
 $= 1\frac{17}{18}$

(2) $1\frac{2}{7} \div 4$
 $= \frac{9}{7} \div \frac{4}{1}$
 $= \frac{9}{7} \times \frac{1}{4}$
 $= \frac{9}{28}$

CHAPTER 16: 2D SHAPES

POLYGON a many sided shape.

In a REGULAR polygon all sides and angles are equal.



pentagon



hexagon



heptagon



octagon



nonagon



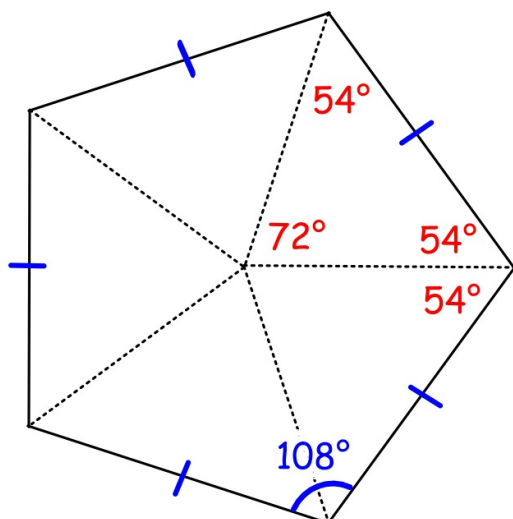
decagon

REGULAR PENTAGON

angle at the centre $360^\circ \div 5 = 72^\circ$

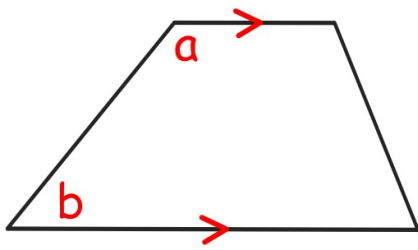
isosceles Δ $180^\circ - 72^\circ = 108^\circ$
 $108^\circ \div 2 = 54^\circ$

interior angle $54^\circ \times 2 = 108^\circ$

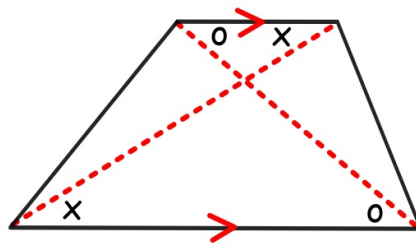


QUADRILATERALS: angle sum 360°

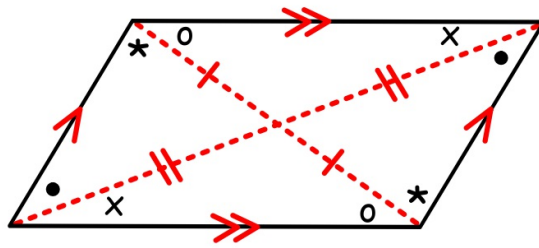
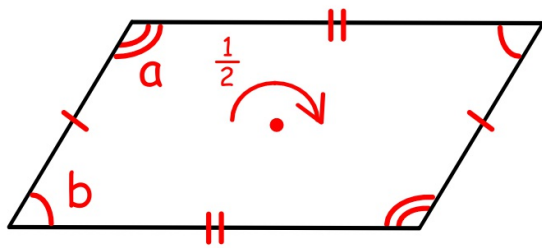
TRAPEZIUM:



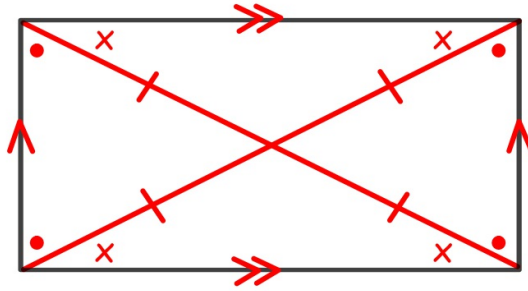
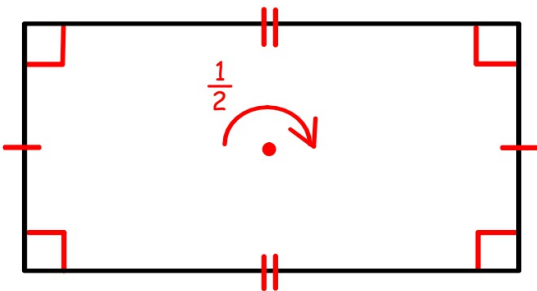
$$a + b = 180^\circ$$



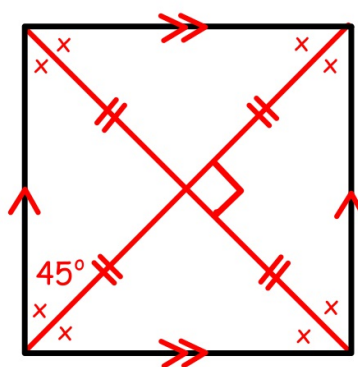
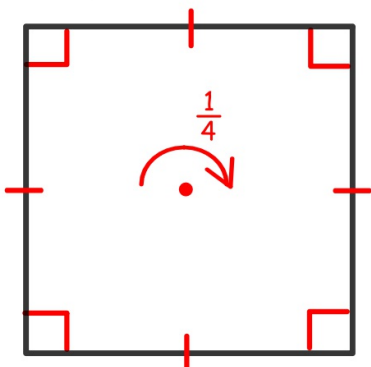
PARALLELOGRAM: a trapezium



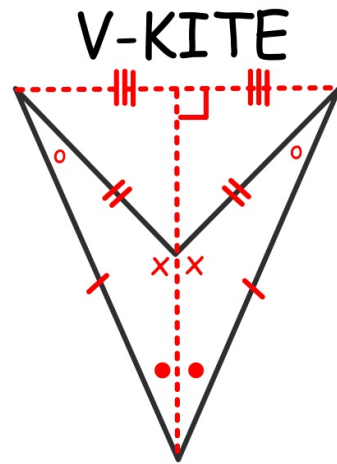
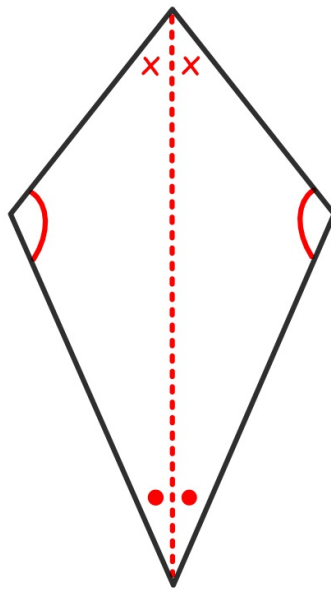
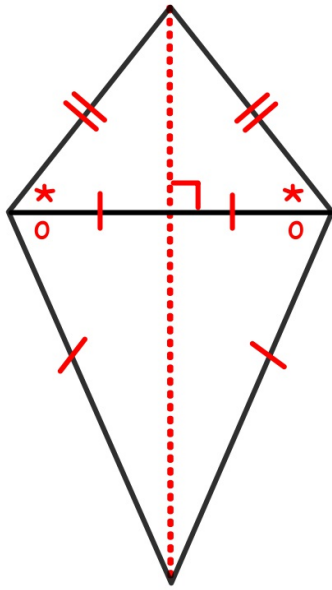
RECTANGLE: a parallelogram



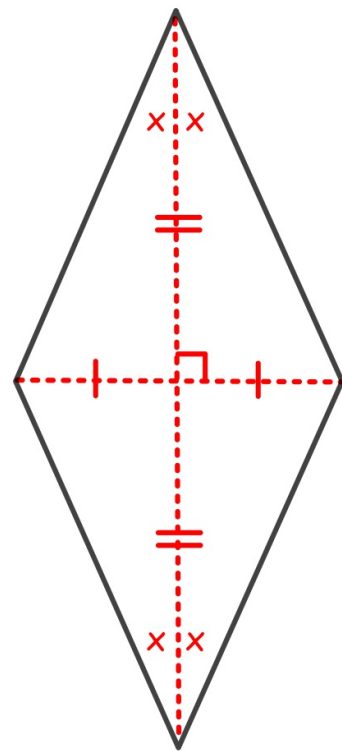
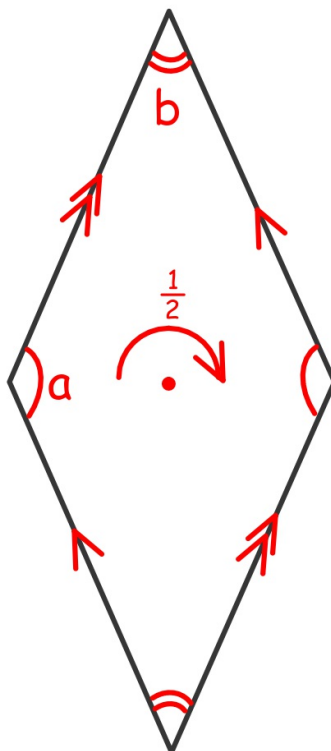
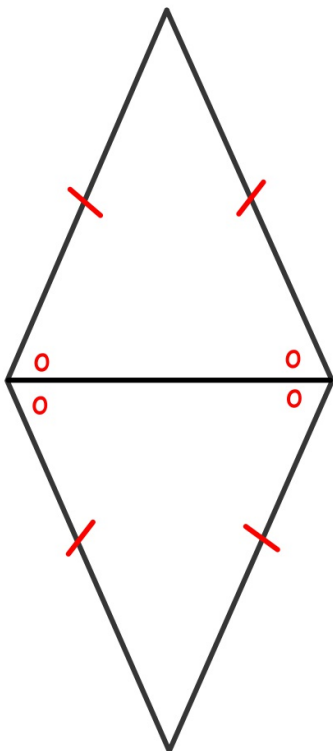
SQUARE: a rectangle



KITE



RHOMBUS a kite and a parallelogram



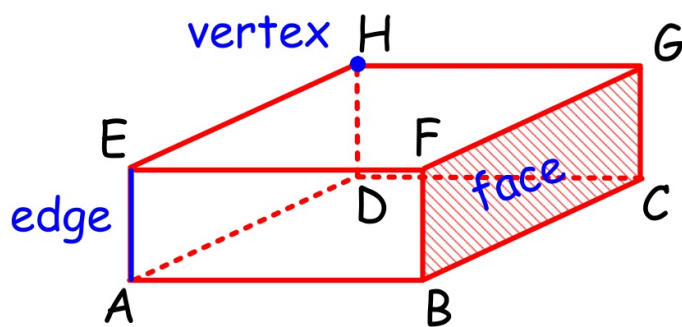
CHAPTER 17: 3D SHAPES

PRISM

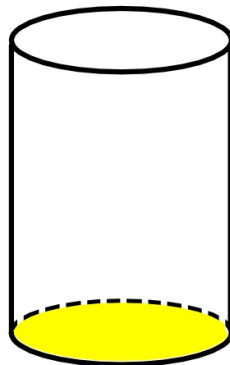
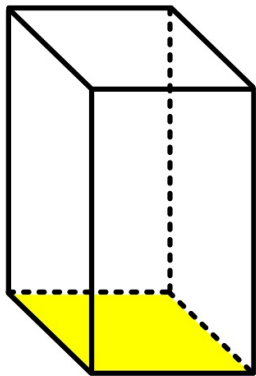
A 3D solid which has the same cross-section throughout its length.

PYRAMID

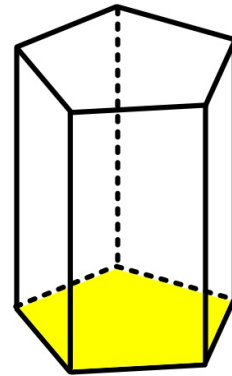
The cross-section reduces to a point.



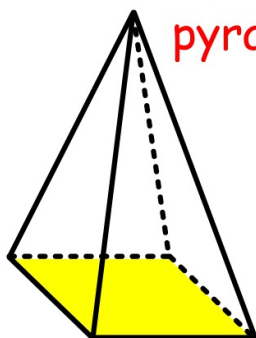
vertex H
edge AE
face BCGF
(not BCFG)



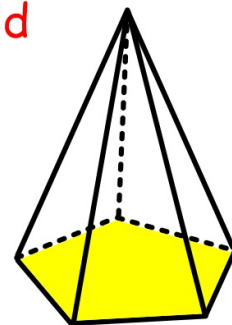
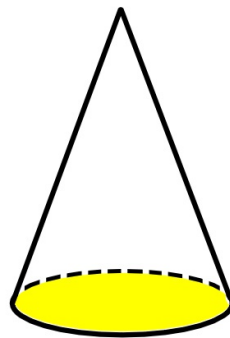
pentagonal prism



square-based pyramid



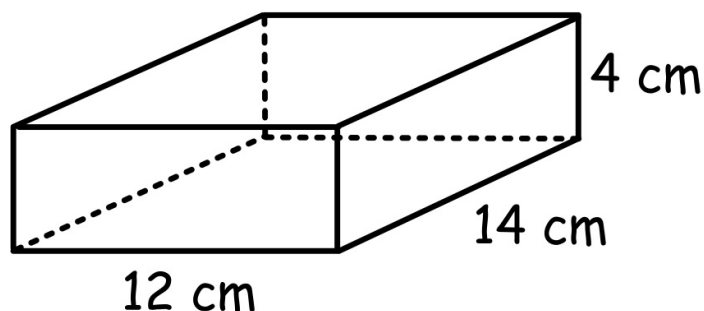
pentagonal-based pyramid



SKELETON MODEL

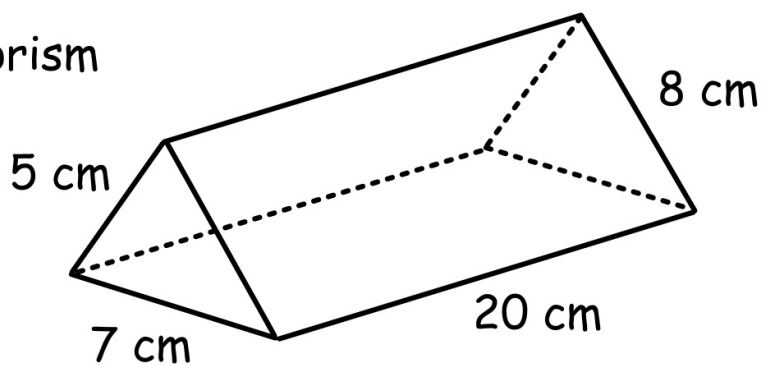
Shows edges - 'hidden' edges dotted.

(1) cuboid



$$\begin{array}{r} \text{total length of edge:} \quad 12 \\ \quad \quad \quad \quad \quad 14 \\ \quad \quad \quad \quad \quad + 4 \\ \hline \quad \quad \quad \quad \quad 30 \end{array} \times 4 = \underline{\underline{120 \text{ cm}}}$$

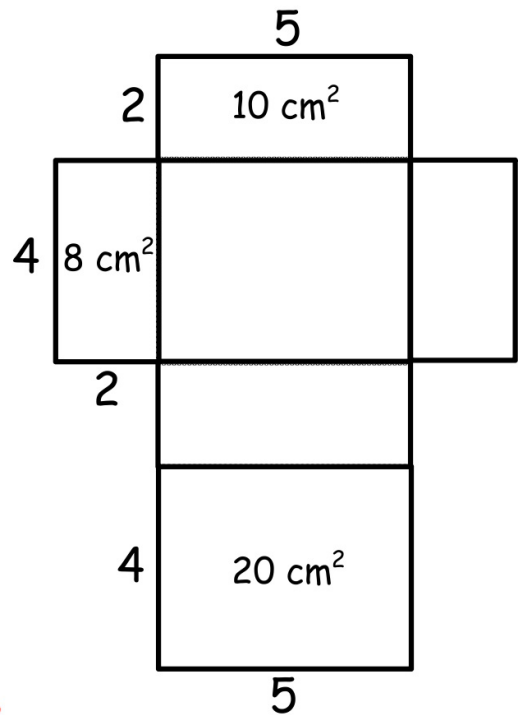
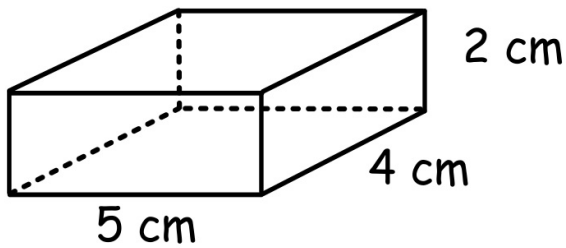
(2) triangular prism



$$\begin{array}{r} \text{total length of edge:} \quad 2 \times 5 = 10 \\ \quad \quad \quad \quad \quad 2 \times 7 = 14 \\ \quad \quad \quad \quad \quad 2 \times 8 = 16 \\ \quad \quad \quad \quad \quad 3 \times 20 = 60 \\ \hline \quad \quad \quad \quad \quad \underline{\underline{100 \text{ cm}}} \end{array}$$

NET Flattened out solid, showing the connected faces.

(1) cuboid



total surface area:

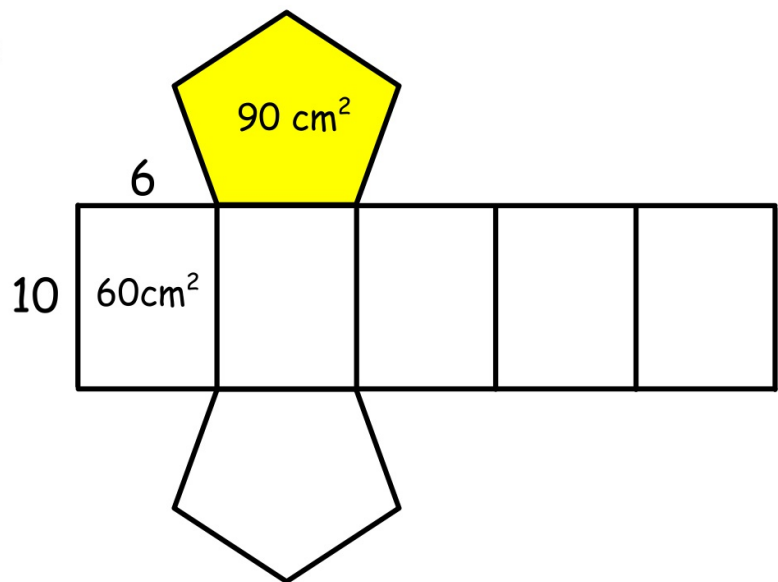
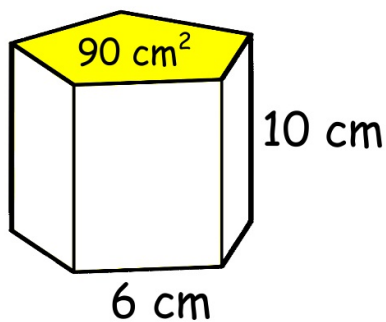
$$5 \times 4 = 20 \text{ cm}^2$$

$$5 \times 2 = 10 \text{ cm}^2$$

$$4 \times 2 = 8 \text{ cm}^2$$

$$\underline{38 \text{ cm}^2} \times 2 = \underline{\underline{76 \text{ cm}^2}}$$

(2) pentagonal prism



total surface area:

$$10 \times 6 = 60 \text{ cm}^2$$

$$5 \times 60 \text{ cm}^2 = 300 \text{ cm}^2$$

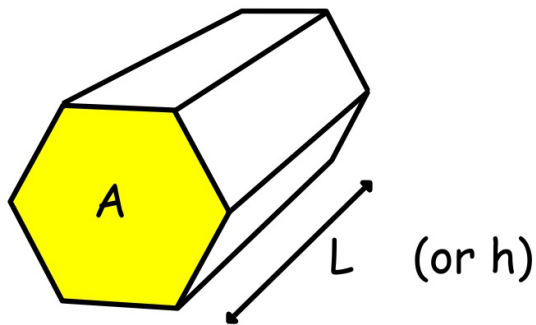
$$2 \times 90 \text{ cm}^2 = \underline{180 \text{ cm}^2}$$

$$\text{total } \underline{\underline{480 \text{ cm}^2}}$$

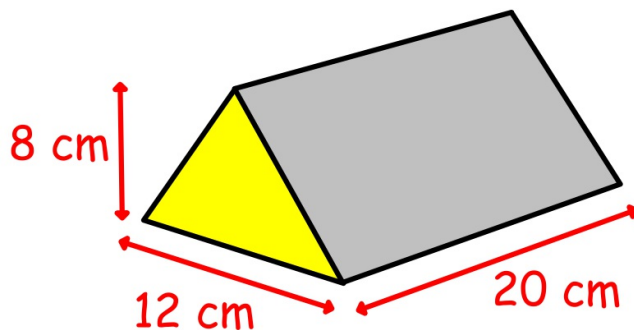
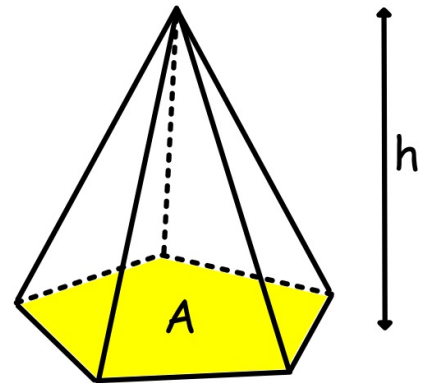
VOLUME OF PRISMS and PYRAMIDS

ensure length units match area units eg. cm with cm^2

PRISM $V = AL$
(or $V = Ah$)

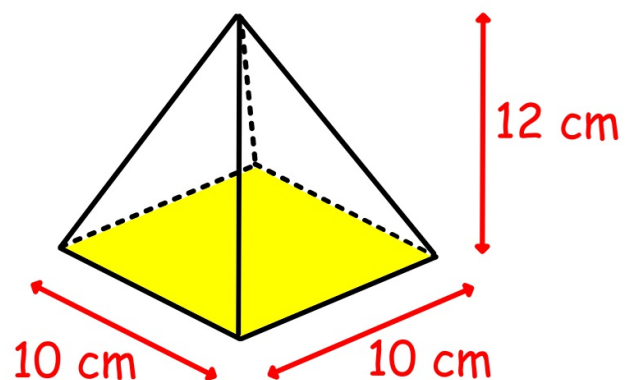


PYRAMID $V = \frac{1}{3} Ah$



$$\begin{aligned} A &= \frac{1}{2} bh \\ &= 12 \times 8 \div 2 \\ &= 48 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} V &= AL \\ &= 48 \times 20 \\ &= 960 \text{ cm}^3 \end{aligned}$$



$$\begin{aligned} A &= lb \\ &= 10 \times 10 \\ &= 100 \text{ cm}^2 \end{aligned}$$

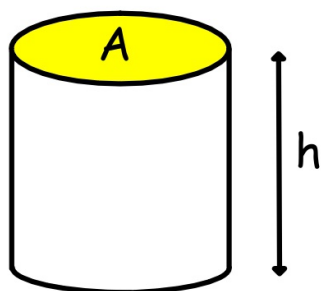
$$\begin{aligned} V &= \frac{1}{3} Ah \\ &= 100 \times 12 \div 3 \\ &= 400 \text{ cm}^3 \end{aligned}$$

CYLINDER and CONE

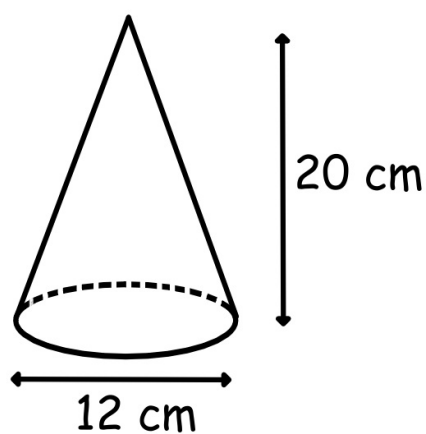
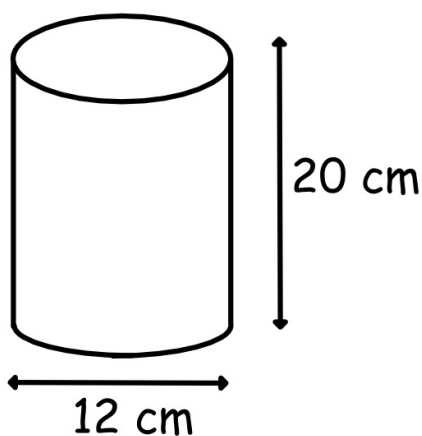
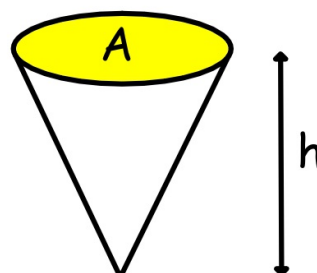
Ensure radius and height units match. eg. cm with cm

$$V = Ah \text{ and } A = \pi r^2$$

$$V = \pi r^2 h$$



$$V = \frac{1}{3} \pi r^2 h$$



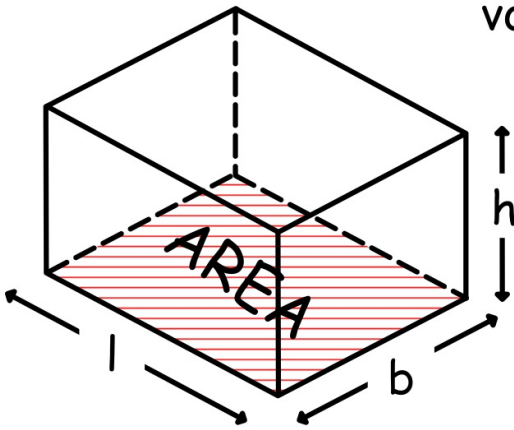
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 6 \times 6 \times 20 \\ &= 2261.946... \\ &\approx \underline{\underline{2260 \text{ cm}^3}} \end{aligned}$$

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \pi \times 6 \times 6 \times 20 \div 3 \\ &= 753.982... \\ &= \underline{\underline{754 \text{ cm}^3}} \end{aligned}$$

CUBOID

$$\text{volume} = \text{length} \times \text{breadth} \times \text{height}$$

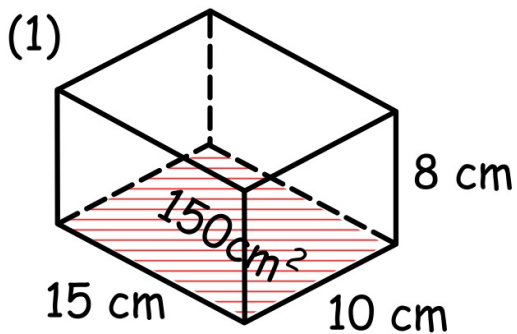
$$\text{volume} = \text{area of base} \times \text{height}$$



$$1000 \text{ cm}^3 = 1 \text{ litre}$$

$$1000 \text{ ml} = 1 \text{ litre}$$

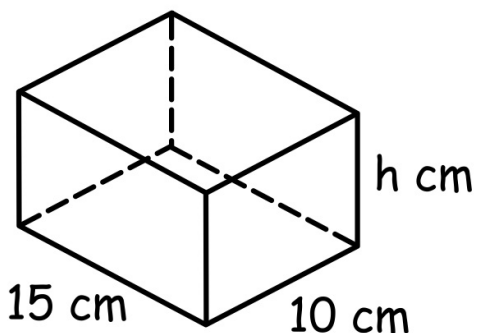
$$1 \text{ ml} = 1 \text{ cm}^3$$



$$\begin{aligned} V &= lbh \\ &= 15 \times 10 \times 8 \\ &= 1200 \text{ cm}^3 \\ &= 1.2 \text{ litres} \end{aligned}$$

$$\begin{aligned} V &= Ah \\ &= 150 \times 8 \\ &= 1200 \text{ cm}^3 \end{aligned}$$

(2) volume 1.2 litres



$$\begin{aligned} V &= lbh \\ 1200 &= 15 \times 10 \times h \\ 1200 &= 150 \times h \\ h &= 1200 \div 150 \\ h &= 8 \end{aligned}$$

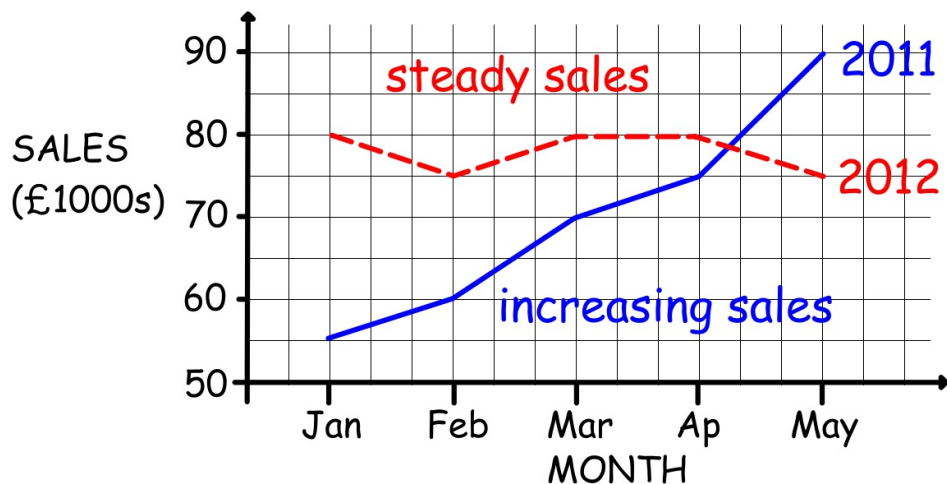
CHAPTER 18: INFORMATION HANDLING 2

CONTINUOUS DATA can take any value

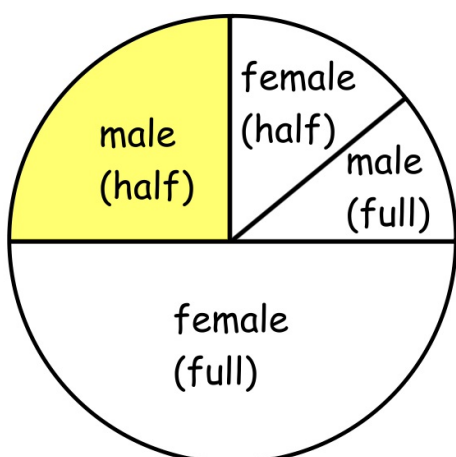
DISCRETE DATA takes particular values

STATISTICAL DIAGRAMS

read information , make comparisons , identify trends



1800 tickets sold.



male(half)

$$\frac{90}{360} \times 1800 = 450 \text{ tickets}$$

Month	Jan	Feb	Mar	April	May
Sales (£1000s)	20	40	30	10	20

PIE CHART

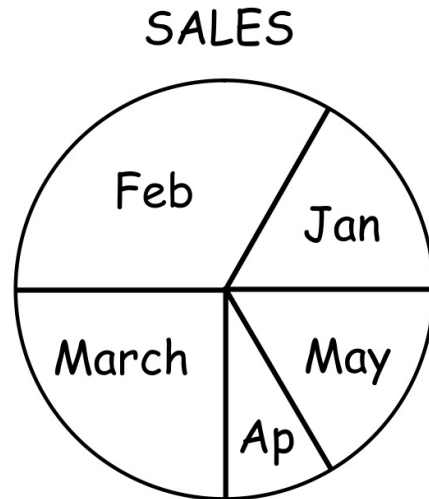
TOTAL SALES = 120

Jan, May $\frac{20}{120} \times 360^\circ = 60^\circ$

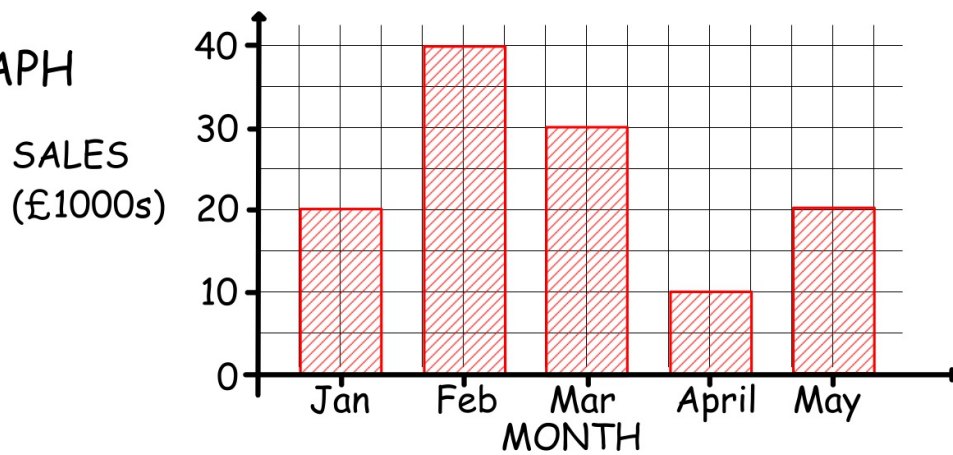
Feb $\frac{40}{120} \times 360^\circ = 120^\circ$

March $\frac{30}{120} \times 360^\circ = 90^\circ$

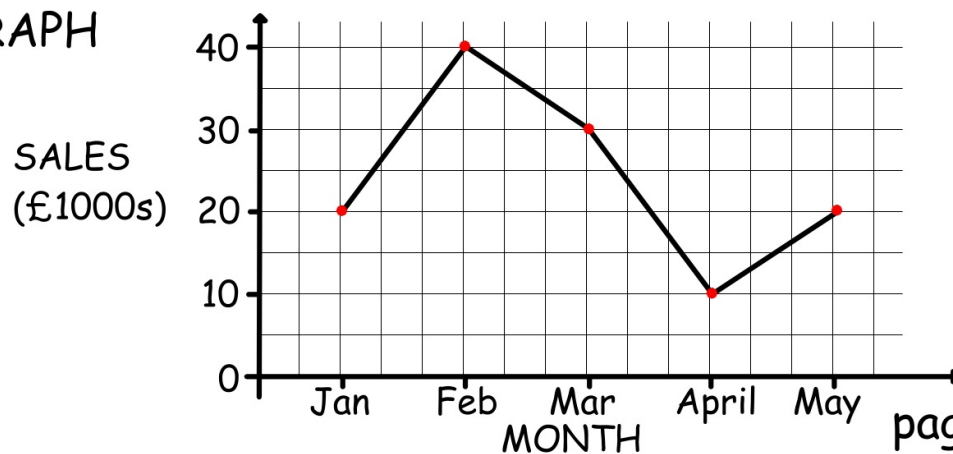
April $\frac{10}{120} \times 360^\circ = 30^\circ$



BAR GRAPH



LINE GRAPH



STEM-AND-LEAF DIAGRAM

Prepare unordered first

5.6 , 3.9 , 6.4 , 4.5 , 3.8 , 5.3 , 6.7 , 3.9 , 5.5 , 4.8 ,
5.0 , 5.8 , 6.2 , 4.2 , 6.1 , 5.3 , 4.9 , 7.3 , 4.4

unordered

```

3 | 9 8 9
4 | 5 8 2 9 4
5 | 6 3 5 0 8 3
6 | 4 7 2 1
7 | 3
    
```

ordered

```

3 | 8 9 9
4 | 2 4 5 8 9
5 | 0 3 3 5 6 8
6 | 1 2 4 7
7 | 3
    
```

n = 19

3|8 = 3.8

BACK-TO-BACK STEM-AND-LEAF

boys					girls				
15	15	21	22	23	11	19	22	25	25
25	26	31	33	34	29	31	34	36	38
37	39	41	46	46	40	46	49	50	50

boys					girls						
		5	5		1		1	9			
	6	5	3	2	1		2	5	5	9	
	9	7	4	3	1		3	1	4	6	8
		6	6	1		4		0	6	9	
						5		0	0		

n = 15

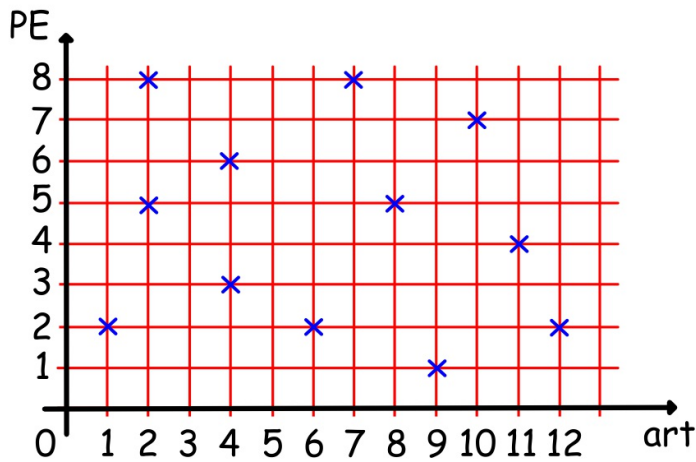
n = 15

1|9 = 1.9

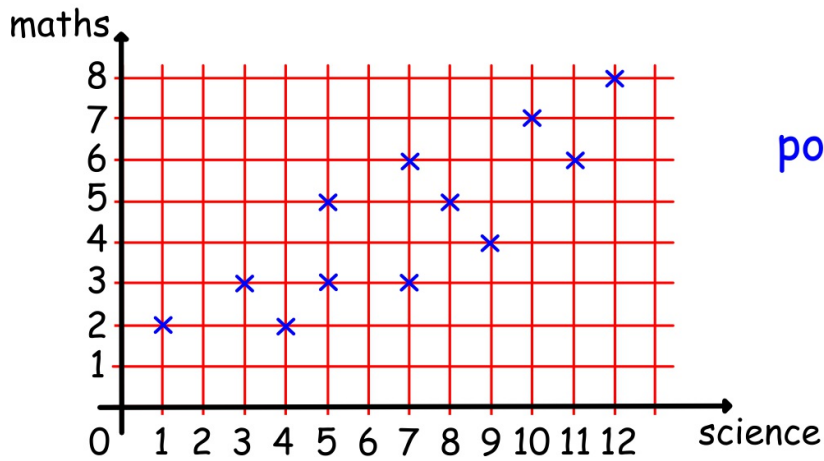
SCATTER DIAGRAMS

If the points plotted lie along a line there is a relationship between the quantities.

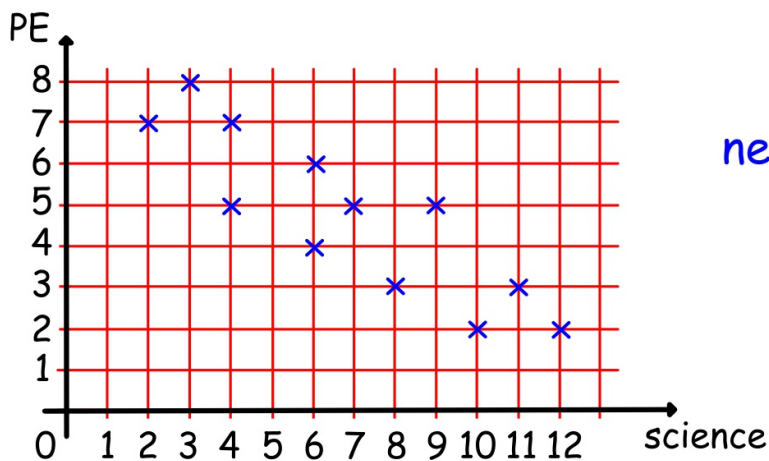
art	2	7	4	8	9	6	11	2	10	1	4	12
PE	8	8	3	5	1	2	4	5	7	2	6	2



no correlation



positive correlation



negative correlation

PROBABILITY

The probability of an event A occurring:

$$P(A) = \frac{\text{number of outcomes involving } A}{\text{number of outcomes possible}}$$

$$0 \leq P \leq 1 \qquad P = 0 \quad \text{impossible}$$

$$P(A) + P(\text{not } A) = 1 \qquad P = 1 \quad \text{certain}$$

Expected outcomes = $P(A) \times$ number of trials

(1) Roll a dice (die) 300 times. How many sixes expected ?

$$\text{number of sixes} = \frac{1}{6} \times 300 = 50$$

(2) choose a letter at random from ARITHMETIC.

$$P(\text{vowel}) = \frac{4}{10} = 0.4$$

$$P(\text{consonant}) = \frac{6}{10} = 0.6 = P(\text{not a vowel})$$

(3) roll two dice , score a total of five.

36 outcomes possible:

(1,1) (1,2) (1,3) (1,6)

(2,1) (2,2) (2,3) (2,6)

· · · ·

· · · ·

(6,1) (6,2) (6,3) (6,6)

4 outcomes total five:

(1,4) (2,3) (3,2) (4,1)

$$P(\text{five}) = \frac{4}{36} = \frac{1}{9}$$