FORMULA for nth TERM

1. If £40000 is invested in a bank at 5% p.a. interest, after n years the amount (£) in the account , A_n , is given by the formula

$$A_n = 40000\,(1.05)^n$$

(a) What is the amount in the account at the end of the first year, and so what was the interest paid in the first year ?

(b) How much would be in the account after two years, and so what would be the interest paid in the second year ?

(c) How many years will it take for the investment to double in value ?

(d) If an interest rate of 6% p.a. is paid on an investment, how many years will it take for the investment to double in value ?

(e) What rate of interest would be required for an investment to double in value after five years ?

2. A new £12000 car will depreciate in value at a rate of 20% every year. After n years the value of the car (£) , V_n , is given by the formula

$$V_n = 12000\,(0.8)^n$$

(a) What is the value of the car after one year , and so what was the loss in value during the first year ?

(b) What is the loss in value during the second year ?

(c) How many years will it take for the car to halve in value ?

(d) As the age of the car increases, what will the value of the car eventually become ?

3. A weedkiller kills 60% of the weeds each time it is applied. After n applications,the percentage of the weeds remaining is given by

$$100\,(0.4)^n$$

(a) How many applications will be required to eliminate over 99% of the weeds ?

(b) What will eventually happen as the applications are continued ?

(c) If the weeds can double their numbers between applications, is the long-term result any different?

(d) A cheaper weedkiller kills only 40% of the weeds each time it is applied, the weeds doubling in number between applications. Can the weeds be eliminated by this weedkiller?

4. A company extracting sand from a quarry estimates that after n years the quantity of sand extracted is given by $80000\,[\,1 - (\,0.5)^n\,]$ tonnes.

It would be uneconomical to extract less than 2500 tonnes per year.

(a) How much sand is extracted in the first year ?

(b) Find how much sand is extracted after two years, and hence the quantity of sand extracted in the second year.

(c) For how long should extraction be carried out ?

(d) If extraction is continued indefinitely, how much sand will be obtained ?

5. **Water leaks from a pool such that after n hours the total loss of water, L_n, is given by the formula**

$$L_n = 40000\,[\,1 - (0.8)^n\,] \text{ gallons}$$

(a) How much water is lost in the first hour ?

(b) What is the total loss of water after three hours ?

(c) How much water is lost in the fourth hour ?

(d) What is the total quantity of water that will be lost from the pool if the leak is not repaired ?

6. **Gold is extracted from a river by "panning". After n pannings the total amount of gold obtained is given by**

$$0.4\,[\,1 - (0.75)^n\,] \text{ Kg.}$$

(a) How much gold is obtained on the first panning ?

(b) How much gold has been obtained after two pannings,
and so how much gold was obtained on the second panning ?

(c) If panning was continued indefinitely, how much gold would be obtained from the river ?

7. **Find the terms U_1, U_2, U_5, U_{10}, U_{20} and state what happens as $n \to \infty$, for the sequences generated by**

(a) $200\,(1.2)^n$

(b) $100\,(0.65)^n$

(c) $4 - (0.85)^n$

(d) $10 - (1.3)^n$

(e) $10\,[\,5 - (1.25)^n\,]$

(f) $10000\,[\,1 - (0.5)^n\,]$

(g) $2 - \dfrac{8}{2^n}$

(h) $\dfrac{2n+1}{2^n}$

(i) $\dfrac{2n^2+1}{n^2}$

SEQUENCES: FORMULA FOR n^{th} TERM

1. If £40000 is invested in a bank at 5% p.a. interest, after n years the amount (£) in the account, A_n, is given by the formula

$$A_n = 40000\,(1.05)^n$$

(a) What is the amount in the account at the end of the first year, and so what was the interest paid in the first year ? £42000 , £2000

(b) How much would be in the account after two years, and so what would be the interest paid in the second year ? £44100 , £2100

(c) How many years will it take for the investment to double in value ? 15 years

(d) If an interest rate of 6% p.a. is paid on an investment, how many years will it take for the investment to double in value ? 12 years

(e) What rate of interest would be required for an investment to double in value after five years ? 15%

2. A new £12000 car will depreciate in value at a rate of 20% every year. After n years the value of the car (£), Vn, is given by the formula

$$V_n = 12000\,(0.8)^n$$

(a) What is the value of the car after one year, and so what was the loss in value during the first year ? £12000 - £9600 = £2400

(b) What is the loss in value during the second year ? £9600 - £7680 = £1920

(c) How many years will it take for the car to halve in value ? 4 years

(d) As the age of the car increases, what will the value of the car eventually become ? £0

3. A weedkiller kills 60% of the weeds each time it is applied.
After n applications, the % of the weeds remaining is given by

$$100\,(0.4)^n$$

(a) How many applications are required to eliminate over 99% of the weeds ?
11

(b) What will eventually happen as the applications are continued ?
no weeds

(c) If the weeds can double their numbers between applications, is the long-term result any different? $100\,(0.8)^n$ no

(d) A cheaper weedkiller kills only 40% of the weeds each time it is applied, the weeds doubling in number between applications.
Can the weeds be eliminated by this weedkiller? $100\,(1.2)^n$ no

4. A company extracting sand from a quarry estimates that after n years the quantity of sand extracted is given by

$$80000\,[\,1 - (\,0.5)^n\,]\text{ tonnes.}$$

It would be uneconomical to extract less than 2500 tonnes per year.

(a) How much sand is extracted in the first year ? 40000 tonnes

(b) Find how much sand is extracted after two years, 60000 , 20000 tonnes
and hence the quantity of sand extracted in the second year.

(c) For how long should extraction be carried out ? 5 years

(d) If extraction is continued indefinitely, how much sand will be obtained ?

80000 tonnes

5. Water leaks from a pool so that after n hours the total loss of water, L_n, is given by the formula

$$L_n = 40000 [1 - (0.8)^n] \text{ gallons}$$

(a) How much water is lost in the first hour ? 8000 gallons

(b) What is the total loss of water after three hours ? 19250 gallons

(c) How much water is lost in the fourth hour ?

23616 - 19250 = 4096 gallons

(d) What is the total quantity of water that will be lost from the pool if the leak is not repaired ? 40000 gallons

6. Gold is extracted from a river by "panning". After n pannings the total amount of gold obtained is given by

$$0.4 [1 - (0.75)^n] \text{ Kg.}$$

(a) How much gold is obtained on the first panning ? 0.1 kg

(b) How much gold has been obtained after two pannings, and so how much gold was obtained on the second panning ?

0.175 - 0.1 = 0.075 kg

(c) If panning was continued indefinitely, how much gold would be obtained from the river ? 0.4 kg

7. Find the terms U_1 , U_2 , U_5 , U_{10} , U_{20} and state what happens as $n \longrightarrow \infty$, for the sequences generated by

(a) $200\,(1.2)^n$ (b) $100\,(0.65)^n$ (c) $4 - (0.85)^n$

(d) $10 - (1.3)^n$ (e) $10\,[\,5 - (1.25)^n\,]$ (f) $10000\,[\,1 - (0.5)^n\,]$

(g) $2 - \dfrac{8}{2^n}$ (h) $2 - \dfrac{2n+1}{2^n}$ (i) $\dfrac{2n^2+1}{n^2}$

(a) 240 , 288 , 497.664 , 1238.34... , 7667.51... $U_n \longrightarrow \infty$

(b) 35 , 42.25 , 11.60... , 1.34... , 0.018... $U_n \longrightarrow 0$

(c) 3.15 , 3.2775 , 3.55... , 3.80... , 3.96... $U_n \longrightarrow 4$

(d) 8.7 , 8.31 , 6.28... , -3.78... , -180.04... $U_n \longrightarrow -\infty$

(e) 37.5 , 34.375 , 19.48... , -43.13... , -817.36... $U_n \longrightarrow -\infty$

(f) 5000 , 7500 , 9687.5 , 9990.23... , 9999.98... $U_n \longrightarrow 10000$

(g) -2 , 0 , 1.75 , 1.992... , 1.999992... $U_n \longrightarrow 2$

(h) 0.5 , 0.75 , 1.65... , 1.979... , 1.99996... $U_n \longrightarrow 2$

(i) 2.5 , 2.25 , 2.03... , 2.0009... , 2.0000009... $U_n \longrightarrow 2$