RECURRENCE RELATIONS

1. The amount of money in a bank account paying 8% p.a. interest is given by the recurrence relation $A_{n+1} = 1.08\,A_n$, where A_n is the amount after n years.

(a) If £10 000 is invested , how much is in the account at the end of the third year and how much interest was paid in the third year ?

(b) How many years will it take for the investment to double in value ?

2. A loan company charges 10% annual interest, paid on the balance outstanding at the start of each year. This is given by the recurrence relation $L_{n+1} = 1.1\,L_n$, where L_n is the loan outstanding at the start of the n^{th}. year.

(a) If a man borrows £20000 , how much does he owe at the end of four years ?

To try to reduce the £20000 loan the man repays £1000 at the end of each year. This is included in the recurrence relation, so $L_{n+1} = 1.1\,L_n - 1000$.

(b) How much will he owe at the end of four years now ? Will he ever repay the loan ?

(c) What will happen if he decides to repay £2000 at the end of each year ?

(d) If he repays £6000 at the end of each year, write the recurrence relation and find the number of years it will take to pay off the entire loan and the last repayment to be made ?

3. A garden pond loses 20% of its water each month through evaporation. This is given by the recurrence relation $V_{n+1} = 0.8\,V_n$, where V_n is the volume of water after n months.

(a) The gardener fills the pond with 1000 gallons of water. How much water will be left after three months ?

(b) How many months will it take for the pond to halve in volume ?

To keep the pond from drying up the gardener adds 150 gallons of water at the end of each month. This is now included in the recurrence relation so that

$$V_{n+1} = 0.8\,V_n + 150$$

(c) How much water is in the pond at the end of the third month now ?

(d) How many months will it take for the pond to halve in volume ? Will it ever empty ?

(e) What is the lowest volume of water, to the nearest gallon, that will be in the pond at any time ?

4. A weedkiller kills 60% of the weeds on each application, but between applications 300 new weeds grow. This is given by the recurrence relation $W_{n+1} = 0.4W_n + 300$, where W_n is the number of weeds after n applications.

Find the lowest number of weeds that can be achieved with this weedkiller.

5. A company wishes to dump 3 tonnes of waste each day in a river. It is known that 60% of any waste is flushed out of the river daily. The local authority decides that the total level of waste in the river must never exceed 6 tonnes at any time.

(a) Write a recurrence relation to model this problem.

(b) Can the company proceed with its plans ?

6. A mushroom farm picks 75% of its mushrooms daily. Each day 6000 new mushrooms are ready for picking.

(a) Write a recurrence relation to model this situation.

(b) In the long-term, what is the number of mushrooms before and after picking ?

7. Each day the streets of a town are swept clear of 70% of the litter found on them. However, each day a further 3.5 tonnes of litter is deposited.

(a) Write an appropriate recurrence relation.

(b) In the long-term, what is the maximum quantity of litter on the streets at any time ?

8. Crop parasites double their numbers each week. Chemical pesticides can be applied once a week to combat them.

TRITOX	**kills 60%**
BITOX	**kills 50%**
MONOTOX	**kills 40%**

(a) Write a recurrence relation for each pesticide.

(b) Describe the effectiveness of each pesticide.

9. During each day of a "flu epidemic" it is found that one third of those who are ill recover. In a particular group of people, eight go down with "flu" each day.

(a) Write a recurrence relation for the situation described.

(b) If this pattern continues for some time, how many people will be ill at any given time ?

10. A drug decays in the body at a rate of 8% during each ten minute period after it is administered.
It is known that to be effective a minimum level of 100 units must be maintained and that a level exceeding 300 units is unsafe.
A doctor prescribes 100 units every hour for a patient.

(a) For each of the first five hours find out whether the drug has been effective ?

(b) If the drug is administered on this basis, will its long-term use be safe ?

SEQUENCES: RECURRENCE RELATIONS

1. The amount of money in a bank account paying 8% p.a. interest is given by the recurrence relation $A_{n+1} = 1.08\, A_n$, where A_n is the amount after n years.

£12597.12 - £11664 = £933.12

(a) If £10 000 is invested , how much is in the account at the end of the third year and how much interest was paid in the third year ?

(b) How many years will it take for the investment to double in value ?

10 years

2. A loan company charges 10% annual interest, paid on the balance outstanding at the start of each year.

This is given by the recurrence relation $L_{n+1} = 1.1\, L_n$, where L_n is the loan outstanding at the start of the $n^{th.}$ year.

(a) If a man borrows £20000, how much does he owe at the end of 4 years ?

£29282

To reduce the £20000 loan, he repays £1000 at the end of each year. This is included in the recurrence relation, so $L_{n+1} = 1.1\, L_n - 1000$.

(b) How much will he owe at the end of four years now ? £24641

Will he ever repay the loan ? no

(c) What will happen if he decides to repay £2000 at the end of each year ?

loan stays at £20000

(d) If he repays £6000 at the end of each year, write the recurrence relation and find the number of years it will take to pay off the entire loan and the last repayment to be made ?

$L_{n+1} = 1.1\, L_n - 6000$

5 years £1579.60

3. A garden pond loses 20% of its water each month through evaporation. This is given by the recurrence relation $V_{n+1} = 0.8\, V_n$, where V_n is the volume of water after n months.

(a) The gardener fills the pond with 1000 gallons of water. How much water will be left after three months ? 512 gallons

(b) How many months will it take for the pond to halve in volume ?
5 months

To keep the pond from drying up the gardener adds 150 gallons of water at the end of each month. This is now included in the recurrence relation so that $V_{n+1} = 0.8\, V_n + 150$

878 gallons
(c) How much water is in the pond at the end of the third month now ?

(d) How many months will it take for the pond to halve in volume ? never
Will it ever empty ? no

(e) What is the lowest volume of water, to the nearest gallon, that will be in the pond at any time ? limit 750, lowest 600 gallons

4. A weedkiller kills 60% of the weeds on each application, but between applications 300 new weeds grow.

This is given by the recurrence relation $W_{n+1} = 0.4W_n + 300$, where W_n is the number of weeds after n applications.

Find the lowest number of weeds that can be achieved with this weedkiller. limit 500, lowest 200

5. A company wishes to dump 3 tonnes of waste each day in a river. It is known that 60% of any waste is flushed out of the river daily. The local authority decides that the total level of waste in the river must never exceed 6 tonnes at any time.

(a) Write a recurrence relation to model this problem. $W_{n+1} = 0.4\, W_n + 6$

(b) Can the company proceed with its plans ? limit 10 ; highest 10,
no, as exceeds 6 tonnes

6. A mushroom farm picks 75% of its mushrooms daily.
Each day 6000 new mushrooms are ready for picking.

(a) Write a recurrence relation to model this situation.
$M_{n+1} = 0.25\ M_n + 6000$

(b) In the long-term, what is the number of mushrooms before and after picking ? limit 8000 ; before 8000 , after 2000

7. Each day the streets of a town are swept clear of 70% of the litter found on them. However, each day a further 3.5 tonnes of litter is deposited.

(a) Write an appropriate recurrence relation. $L_{n+1} = 0.3\ L_n + 3.5$

(b) In the long-term, what is the maximum quantity of litter on the streets at any time ? limit 5 ; max 5 tonnes (before sweeping)

8. Crop parasites double their numbers each week.
Chemical pesticides can be applied once a week to combat them.

TRITOX	kills 60%	$T_{n+1} = 0.8\ T_n$	eliminates
BITOX	kills 50%	$B_{n+1} = 1\ B_n$	controls
MONOTOX	kills 40%	$M_{n+1} = 1.2\ M_n$	fails

(a) Write a recurrence relation for each pesticide.

(b) Describe the effectiveness of each pesticide.

9. During each day of a “flu epidemic” it is found that one third of those who are ill recover. In a particular group of people, eight go down with “flu” each day.
$I_{n+1} = {}^{2}/_{3}\ I_n + 8$

(a) Write a recurrence relation for the situation described.

(b) If this pattern continues for some time, how many people will be ill at any given time ?
max 24

10. A drug decays in the body at a rate of 8% during each ten minute period after it is administered. $L_{n+1} = (0.92)^6 L_n + 100$

It is known that to be effective a minimum level of 100 units must be maintained and that a level exceeding 300 units is unsafe.

A doctor prescribes 100 units every hour for a patient.

(a) For each of the first five hours, find out whether the drug has been effective ? effective from 3rd hour

(b) If the drug is administered on this basis, will its long-term use be safe ?

limit 254.0359... ; max 254.0359... , min 154.0359..

in range 100-300 units, so safe and effective

						min level before dose
$L_0 =$ 100						
$L_1 = (0.92)^6 \times 100$	+	100	=	160.63..		60.63... < 100
$L_2 = (0.92)^6 \times 160.63...$	+	100	=	197.40..		97.40... < 100
$L_3 = (0.92)^6 \times 197.40...$	+	100	=	219.69..		119.69...
$L_4 = (0.92)^6 \times 219.69...$	+	100	=	233.21..		133.21...
$L_5 = (0.92)^6 \times 233.21..$	+	100	=	241.41..		141.41...