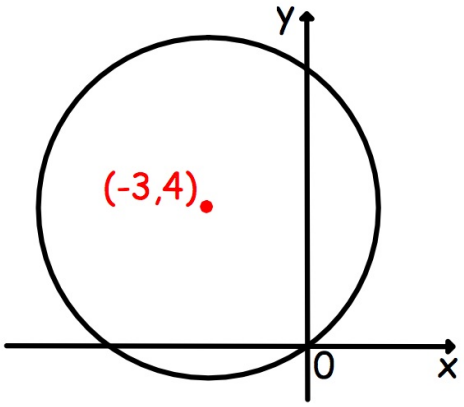
# CIRCLES

**(1)** Find the equation of the circle with centre (-3,4) and passing through the origin.

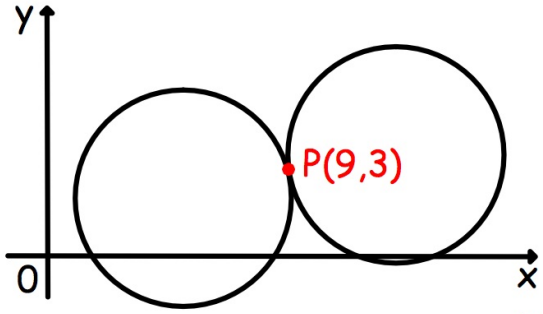



(2)

**(2)** Two identical circles touch at the point P(9,3) as shown.

One of the circles has equation
$x^2 + y^2 - 10x - 4y + 12 = 0$

Find the equation of the other circle.

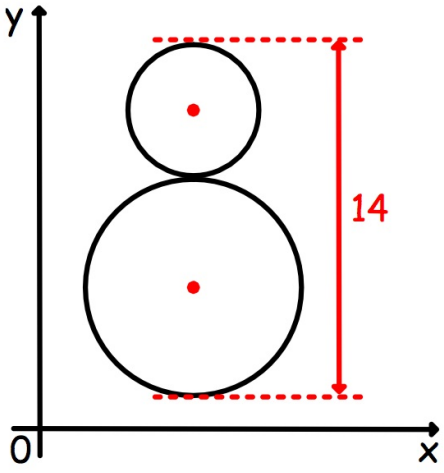



(5)

**(3)** A bakery makes gingerbread men each 14cm high with a circular "head" and "body".

The equation of the body is
$x^2 + y^2 - 10x - 12y + 45 = 0$
and the line of centres is parallel to the Y-axis.

Find the equation of the "head".

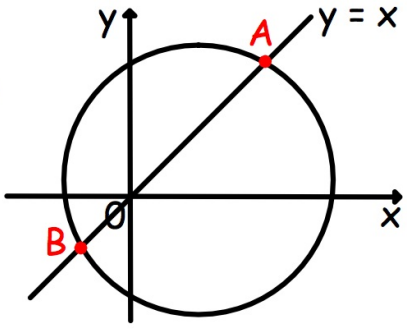



(5)

**(4)** The line $y = x$ cuts the circle $x^2 + y^2 - 6x - 2y - 24 = 0$ at points A and B.

(a) Find the coordinates of A and B. (3)

(b) Find the equation of the circle which has AB as diameter. (3)



(5) Find the equation of the tangent at the point (3,4) on the circle $x^2 + y^2 + 2x - 4y - 15 = 0$. (4)

(6) Find the equation of the tangent at the point (3,1) on the circle $x^2 + y^2 - 4x + 6y - 4 = 0$. (5)

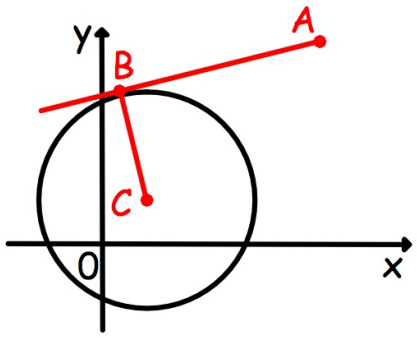
(7) Explain why the equation $x^2 + y^2 + 2x + 3y + 5 = 0$ does NOT represent a circle. (2)

(8) For what range of values of $k$ does the equation $x^2 + y^2 - 6x + 4y + k = 0$ represent a circle ? (3)

(9) AB is a tangent at B to the circle with centre C and equation $(x - 2)^2 + (y - 2)^2 = 25$

The point A has coordinates (10,8).

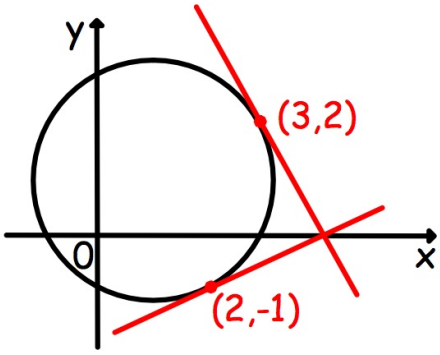
Find the EXACT area of triangle ABC. (5)




(10) The circle shown has equation $(x - 1)^2 + (y - 1)^2 = 5$

Tangents are drawn at the points (3,2) and (2,-1).

Show that the tangents are perpendicular to each other. (4)




(11) Find the possible values of $k$ for which $x - y = k$ is a tangent to the circle $x^2 + y^2 = 18$. (5)

**1.**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 2.4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 2 | 2.4 | | | | | 2 | | 2.4.3 | | Source: **1999 P1 qu.4** |

- $\bullet^1$ $r^2 = 25$ stated or implied by $\bullet^2$.
- $\bullet^2$ $(x+3)^2 + (y-4)^2 = 25$



**2.**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 2.4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 5 | 2.4 | | | | | 5 | | 2.4.2 | (3.1.6) | Source: **1997 P1 qu.12** |

- $\bullet^1$ use P as midpoint of $C_1C_2$
- $\bullet^2$ $C_1 = (5,2)$
- $\bullet^3$ $C_2 = (13,4)$
- $\bullet^4$ radius $= \sqrt{17}$
- $\bullet^5$ $(x-13)^2 + (y-4)^2 = 17$



**3.**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 2.4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 5 | 2.4 | | | | | 5 | | 2.4.2 | 2.4.3 | Source: **1990 P1 qu.7** |

- $\bullet^1$ centre of body $= (5,6)$
- $\bullet^2$ radius of body $= 4$
- $\bullet^3$ radius of head $= 3$
- $\bullet^4$ centre of head $= (5,13)$
- $\bullet^5$ $(x-5)^2 + (y-13)^2 = 9$

**4.**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 2.4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (a) | 3 | 2.4 | | | | | 3 | | 2.4.4 | | Source: **1994 P1 qu.8** |
| (b) | 3 | 2.4 | | | | | 3 | | 2.4.3 | | |

- $\bullet^1$ $x^2 + x^2 - 6x - 2x - 24 = 0$
- $\bullet^2$ $(x+2)(x-6) = 0$
- $\bullet^3$ $(-2,-2)$ **and** $(6,6)$

*OR*

- $\bullet^4$ centre is $(2,2)$
- $\bullet^5$ radius is $\sqrt{32}$ or equivalent
- $\bullet^6$ $(x-2)^2 + (y-2)^2 = 32$

5.

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 1.1 Source |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 4 | 1.1 | | | | | 4 | | 1.1.1 | 1.1.9, 2.4.2 | **1996 P1 qu.4** |

- •1 centre $= (-1,2)$
- •2 $m_{radius} = \frac{1}{2}$
- •3 $m_{tgt} = -2$
- •4 $y-4=-2(x-3)$



6.

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 2.4 Source |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 5 | 2.4 | | | | | 5 | | 2.4.4 | | **1991 P1 qu.8** |

- •1 *strat*: use centre and tgt $\perp$ radius
- •2 *centre* $= (2,-3)$
- •3 $m_{radius} = 4$
- •4 $m_{tgt} = -\frac{1}{4}$
- •5 $y-1=-\frac{1}{4}(x-3)$



7.

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 2.4 Source |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 2 | 2.4 | | | | | 2 | | 2.4.2 | | **1993 P1 qu.18** |

- •1 $g^2+f^2-c=-1\frac{3}{4}$
- •2 $r=\sqrt{-1\frac{3}{4}}$ which is not possible



8.

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 2.4 Source |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 2.4 | | | | | 2 | 1 | 2.4.2 | | **1997 P1 qu.14** |

- •1 $g^2+f^2-c>0$
- •2 $r^2=9+4-c$
- •3 $c<13$

9.

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 2.4 Source |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 5 | 2.4 | | | | | 5 | | 2.4.1 | 1.1.2, 0.1 | **1992 P1 qu.16** |

- •1 *strat*: *i.e* find AC then AB
- •2 centre = (2, 2) **and** radius = 5
- •3 $AC = 10$
- •4 $AB = \sqrt{75}$ units
- •5 area $= \frac{25}{2}\sqrt{3}$ square units



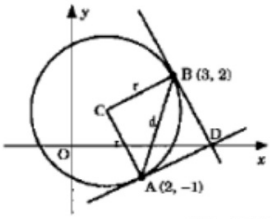
10.

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 2.4 Source |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 4 | 2.4 | | | | | 4 | | 2.4.1 | 1.1.9 | **1994 P1 qu.5** |

- •1 *centre* = (1, 1)
- •2 $m_{radii} = \frac{1}{2}, -2$
- •3 $m_{tgts} = -2, \frac{1}{2}$
- •4 $-2 \times \frac{1}{2} = -1 \Rightarrow$ tgts are $\perp$

***OR***

- •1 *centre* = (1, 1)
- •2 $r = \sqrt{5}$, $d = \sqrt{10}$
- •3 Show $A\hat{C}B = 90^\circ$
- •4 State tangents $\perp$ to radii




11.

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 2.4 Source |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 5 | 2.4 | | | | | 2 | 3 | 2.4.4 | | **1989 P1 qu.18** |

- •1 $x^2 + (x-k)^2 = 18$
- •2 $2x^2 - 2kx + k^2 - 18 = 0$
- •3 *strat*: "$b^2 - 4ac$" $= 0$
- •4 $(-2k)^2 - 4.2.\left(k^2 - 18\right)$
- •5 $k = \pm 6$

(SQA H 2011)

Circle $C_1$ has equation $(x+1)^2 + (y-1)^2 = 121$ (-1,1) r = 11

A circle $C_2$ with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside $C_1$.

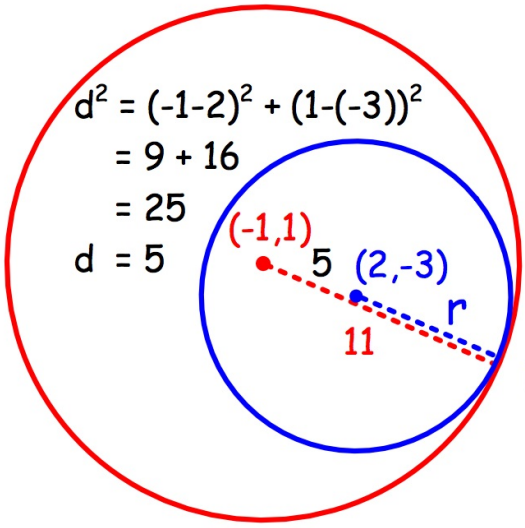
The circles have no points of contact.

What is the range of values of p ? 9

$g = -2 \quad f = 3 \quad c = p$

(2,-3)

$r = \sqrt{g^2 + f^2 - c} = \sqrt{13 - p}$




$r < 6$

$13 - p > 0 \quad 13 - p < 36$

$-p > -13 \quad -p < 23$

$p < 13 \quad p > -23$

$-23 < p < 13$

touching $r = 11 - 5 = 6$

not touching $r < 6$

(SQA H 2011)

Circle $C_1$ has equation $(x+1)^2 + (y-1)^2 = 121$

A circle $C_2$ with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside $C_1$.

The circles have no points of contact.

What is the range of values of p ? 9

| | Generic Scheme | | Illustrative Scheme | |
|---|---|---|---|---|
| 7 | | | | |
| •1 | ic | state centre of $C_1$ | •1 (–1, 1) | |
| •2 | ic | state radius of $C_1$ | •2 11 | Do not accept $\sqrt{121}$ |
| •3 | ic | state centre of $C_2$ | •3 (2, –3) | |
| •4 | pd | find radius of $C_2$ in terms of $p$ | •4 $\sqrt{13-p}$ | Accept $c$ in lieu of $p$ |
| •5 | ic | interpret upper bound for $p$ | •5 $p < 13$ | |
| •6 | ic | find distance between centres ($d$) | •6 5 | **stated explicitly** |
| •7 | ss | identify relevant relationship | •7 $\sqrt{13-p} < 6$ or $r_2 + d < 11$ or $r_2 < 6$ | |
| •8 | ic | develop relationship by squaring | •8 $13 - p < 36$ | |
| •9 | pd | find lower bound for $p$ | •9 $p > -23$ | |

9