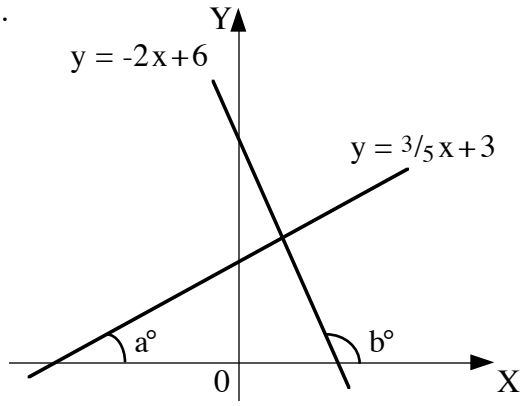
HOME EXERCISE 2: SOLUTIONS

1.



(a) Calculate the angles a° and b° that the lines shown make with the positive OX direction. (4)

(b) Hence calculate the angle between the two lines. (1)

(a)

$y = \frac{3}{5}x + 3$ $\quad$ $y = -2x + 6$

$y = mx + C$ $\quad$ $y = mx + C$

$m = \frac{3}{5}$ $\quad$ $m = -2$

$\tan a° = \frac{3}{5}$ $\quad$ $\tan b° = -2 \quad \left(\tan^{-1} 2\right)$

$a = \tan^{-1} \frac{3}{5}$ $\quad$ $b = 180 - 63 \cdot 434...$

$a = 30 \cdot 963...$ $\quad$ $b = 116 \cdot 565...$

$a° = 31 \cdot 0°$ $\quad$ $b° = 116 \cdot 6°$

(b)

$b° - a° = 116 \cdot 565... - 30 \cdot 963... = 85 \cdot 601... = 85 \cdot 6°$

2. Given that the lines with equations $3x - 4y + 12 = 0$ and $y = ax - 6$ are perpendicular, find the value of a. (2)

rearrange for $y = mx + C$

$3x - 4y + 12 = 0$

$3x + 12 = 4y$

$\frac{3}{4}x + 3 = y$

$y = \frac{3}{4}x + 3$

$y = \frac{3}{4}x + 3$ $\quad$ $y = ax - 6$

$y = mx + C$ $\quad$ $y = mx + C$

$m_1 = \frac{3}{4}$ $\quad$ $m_2 = a$

for perpendicular lines

$m_1 \cdot m_2 = -1$

$\frac{3}{4} \cdot a = -1$

$a = -\frac{4}{3}$

3. If $f(x) = 2x - x^2$ and $g(x) = x + 1$

(a) write in simplest form $f(g(x))$ (3)

(b) If $h(x) = \dfrac{1}{f(g(x))}$, state the values of x for which the function $h(x)$ is undefined. (2)

(a)

$f(g(x))$

$= f(x+1)$

$= 2(x+1) - (x+1)^2$

$= 2(x+1) - (x^2 + 2x + 1)$

$= 2x + 2 - x^2 - 2x - 1$

$= 1 - x^2$

(b)

$h(x) = \dfrac{1}{f(g(x))}$

$= \dfrac{1}{1 - x^2}$

$= \dfrac{1}{(1+x)(1-x)}$

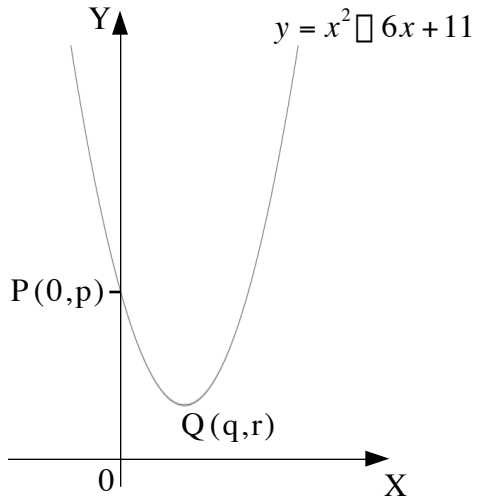
$h(x)$ undefined where

$(1+x)(1-x) = 0$

$(1+x) = 0$ or $(1-x) = 0$

$x = -1$ or $x = 1$

4.



The graph of the function $f(x) = x^2 - 6x + 11$ is shown.

(a) Write $x^2 - 6x + 11$ the form $(x + a)^2 + b$. (2)

(b) The curve meets the y-axis at point P(0,p) and the turning point is Q(q,r).
Write the values of p, q and r. (3)

(c) If $g(x) = 2 - f(x)$, sketch the graph of $g(x)$, marking clearly the turning point and the points where the graph meets the axes. (3)

(a)

$x^2 - 6x + 11$
$= x^2 - 6x + 9 - 9 + 11$
$= (x - 3)^2 - 9 + 11$
$= (x - 3)^2 + 2$

(b)

$f(x) = x^2 - 6x + 11$
$f(0) = 0^2 - 6 \times 0 + 11 = 11$

$p = 11$

minimum at $x = 3$, $q = 3$

$f(3) = (x - 3)^2 + 2$
$= (3 - 3)^2 + 2$
$= 0^2 + 2$
$= 2$ $r = 2$

(c)