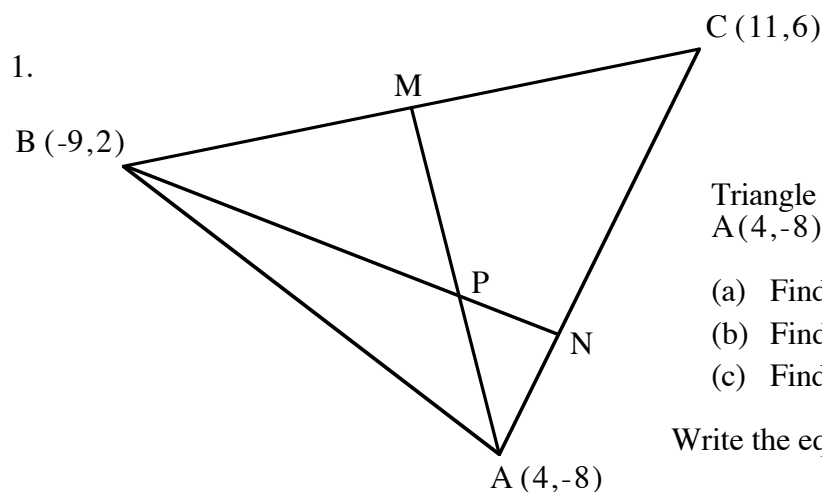


# HOME EXERCISE 3: SOLUTIONS



Triangle ABC shown has vertices A(4, -8), B(-9, 2) and C(11, 6).

- (a) Find the equation of the **median** AM. (3)
- (b) Find the equation of the **altitude** BN. (4)
- (c) Find their point of intersection P. (2)

Write the equations in the form  $Ax + By + C = 0$ .

(a) **median** AM

midpoint of BC

$$\left[ \frac{-9 + 11}{2}, \frac{2 + 6}{2} \right]$$

$$\left[ \frac{2}{2}, \frac{8}{2} \right]$$

$$M_{BC} (1, 4)$$

gradient of AM

$$m_{AM} = \frac{8 - 4}{4 - 1}$$

$$= \frac{4}{3}$$

$$= 4$$

$$y - b = m(x - a)$$

$$y - 8 = 4(x - 4)$$

$$y + 8 = 4x + 16$$

$$4x + y - 8 = 0$$

(b) **altitude** BN

gradient of AC

$$m_{AC} = \frac{6 - (-8)}{11 - 4}$$

$$= \frac{14}{7}$$

$$= 2$$

perpendicular gradient

$$m_1 \cdot m_2 = -1$$

$$m_{BN} = -\frac{1}{2}$$

$$y - b = m(x - a)$$

$$y - 2 = -\frac{1}{2}(x - (-9))$$

$$2y - 4 = -1(x + 9)$$

$$2y - 4 = -x - 9$$

$$x + 2y + 5 = 0$$

(c) point of intersection P

$$4x + y - 8 = 0 \quad \times 2$$

$$x + 2y + 5 = 0 \quad \times (-1)$$

$$8x + 2y - 16 = 0$$

$$-x - 2y - 5 = 0$$

$$\text{add } 7x - 21 = 0$$

$$7x = 21$$

$$x = 3$$

$$4x + y - 8 = 0$$

$$4 \cdot 3 + y - 8 = 0$$

$$12 + y - 8 = 0$$

$$y + 4 = 0$$

$$y = -4$$

$$P (3, -4)$$

2. If  $f(x) = \frac{1}{x}$ ,  $x \neq 0$  and  $g(x) = \frac{x}{1-x}$ ,  $x \neq 1$

(a) write in simplest form: (i)  $f(g(x))$  (2)

(ii)  $g(f(x))$  (2)

(b) explain why function  $f(x)$  is its own inverse. (1)

(a) (i)

$$f(g(x)) = f\left(\frac{x}{1-x}\right)$$

$$= \frac{1}{\frac{x}{1-x}}$$

$$= \frac{1-x}{x}$$

(ii)

$$g(f(x)) = g\left(\frac{1}{x}\right)$$

$$= \frac{\frac{1}{x}}{1-\frac{1}{x}}$$

$$= \frac{\frac{1}{x} \cdot x}{x-1}$$

$$= \frac{1}{x-1}$$

(b)

$$f(f(x)) = f\left(\frac{1}{x}\right)$$

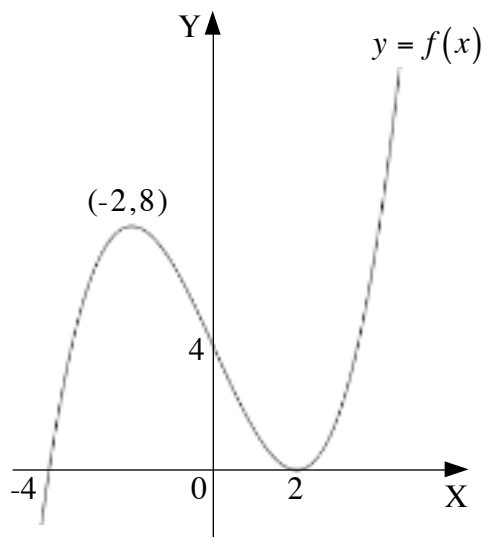
$$= \frac{1}{\frac{1}{x}}$$

$$= x$$

since  $f(f(x)) = x$

function  $f$  is its own inverse

3. The graph of  $y = f(x)$  shown has turning points at  $(-2, 8)$  and  $(2, 0)$ .  
The graph meets the axes at the points  $(-4, 0)$ ,  $(2, 0)$  and  $(0, 4)$ .



For each part (a), (b) and (c) below make a **neat** sketch of the graph required.  
Annotate the graphs with the images of the four points given on the graph  $y = f(x)$ .

- (a)  $y = f(x) - 4$ . (2)  
 (b)  $y = f(x + 2)$ . (2)  
 (c)  $y = -f(x)$ . (2)

