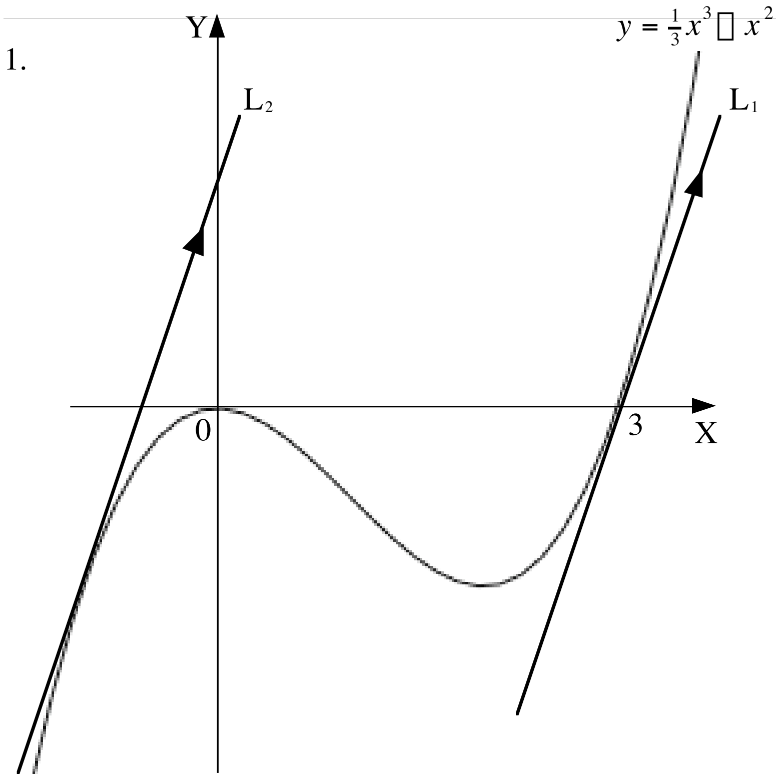
HOME EXERCISE 4: SOLUTIONS

1.



The graph with equation $y = \frac{1}{3}x^3 - x^2$ is shown.

(a) Line L$_1$ is a tangent to the graph at the point $(3,0)$.

Find the equation of line L$_1$. (3)

(b) Line L$_2$ is also a tangent to the graph and is parallel to line L$_1$.

Find the x-coordinate of the point where line L$_2$ meets the curve. (2)

(a)

$$f(x) = \tfrac{1}{3}x^3 - x^2$$
$$f'(x) = \tfrac{1}{3} \times 3x^2 - 2x$$
$$= x^2 - 2x$$
$$f'(3) = 3^2 - 2 \times 3$$
$$= 9 - 6$$
$$= 3$$
$$m = 3$$

$$m = 3 \quad \left(\overset{a}{3}, \overset{b}{0}\right)$$
$$y - b = m(x - a)$$
$$y - 0 = 3(x - 3)$$
$$y = 3x - 9$$

(b)

$$f'(x) = x^2 - 2x\,,\ m = 3$$
$$x^2 - 2x = 3$$
$$x^2 - 2x - 3 = 0$$
$$(x+1)(x-3) = 0$$
$$x + 1 = 0 \text{ or } x - 3 = 0$$
$$x = -1 \text{ or } x = 3$$
for line L$_2$ $\quad x = -1$

2. (a) Write $3x^2 + 12x + 20$ the form $a(x+b)^2 + c$. (3)

(b) Hence state the minimum value of $3x^2 + 12x + 20$ and the corresponding value of *x*. (2)

(a)

$$3x^2 + 12x + 20$$
$$= 3(x^2 + 4x) + 20$$
$$= 3(x^2 + 4x + 4 - 4) + 20$$
$$= 3(x^2 + 4x + 4) - 12 + 20$$
$$= 3(x+2)^2 + 8$$

(b)

when $x = -2$
$$3(x+2)^2 + 8$$
$$= 3 \times (-2+2)^2 + 8$$
$$= 3 \times 0^2 + 8$$
$$= 8$$

minimum value 8
when $x = -2$

3. (a) Solve the equation $2\sin x^{\circ} + \sqrt{3} = 0$, $0 \le x \le 360$. (3)

(b) Hence solve the equation $2\sin(2x - 10)^{\circ} + \sqrt{3} = 0$, $0 \le x \le 180$. (2)

(a)

$$2\sin x^{\circ} + \sqrt{3} = 0$$

$$2\sin x^{\circ} = -\sqrt{3}$$

$$\sin x^{\circ} = -\frac{\sqrt{3}}{2}$$

$$x = 240 \text{ , } 300$$

A, S, T, C is where functions are positive:

sin +	S	A	sin +
180 - a = 120°		a = sin^{-1} $\sqrt{3}/2$ = 60°	
180 + a = 240°		360 - a = 300°	
sin -	T	C	sin -

(b)

$$2x - 10 = 240 \text{ , } 300$$

$$2x = 250 \text{ , } 310$$

$$x = 125 \text{ , } 155$$

4. If $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+1}$, $x \ge -1$

(a) write in simplest form: (i) $f(g(x))$ (2)

(ii) $g(f(x))$. (2)

(b) Comment on the results of part (a) regarding functions f and g. (1)

(a) (i)

$$f(g(x)) = f\left(\sqrt{x+1}\right)$$

$$= \left(\sqrt{x+1}\right)^2 - 1$$

$$= x + 1 - 1$$

$$= x$$

(ii)

$$g(f(x)) = g\left(x^2 - 1\right)$$

$$= \sqrt{x^2 - 1 + 1}$$

$$= \sqrt{x^2}$$

$$= x$$

(b)

since $f(g(x)) = g(f(x)) = x$

f and g are inverse functions