HOME EXERCISE 5: SOLUTIONS

1.

$$y = \tfrac{1}{3}x^3 - x^2$$
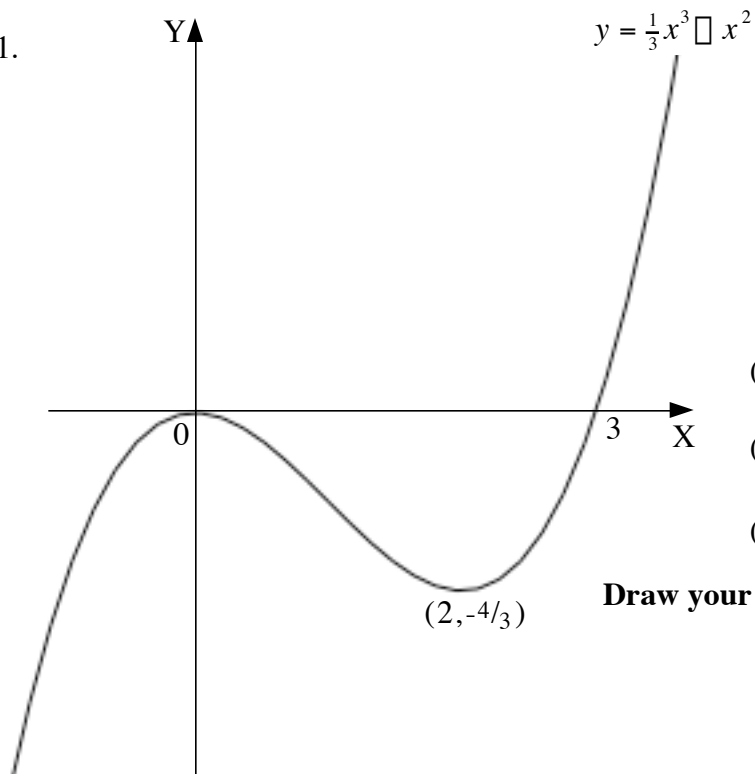
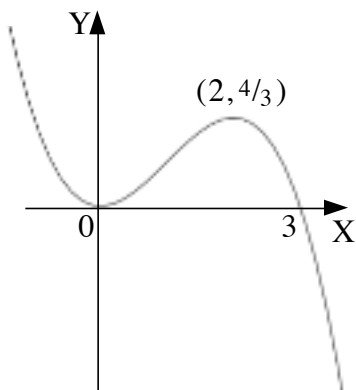The graph with equation
$y = \tfrac{1}{3}x^3 - x^2$ is shown.

The graph meets the axes at $(0,0)$
and $(3,0)$ and has stationary
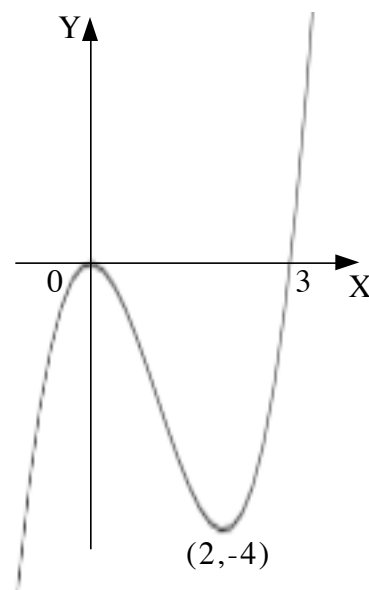points $(0,0)$ and $(2,-4/3)$.

Use this graph to sketch the graph:

(a) with equation $y = x^2 - \tfrac{1}{3}x^3$ (2)

(b) with equation $y = x^3 - 3x^2$ (2)

(c) of the derived function.   (2)

Draw your sketches on three separate diagrams.

$(2,-4/3)$

(a)

$$y = x^2 - \tfrac{1}{3}x^3 = -\left(\tfrac{1}{3}x^3 - x^2\right)$$

$y = -f(x)$, reflection in x - axis

$(2,4/3)$

(b)

$$y = x^3 - 3x^2 = 3\left(\tfrac{1}{3}x^3 - x^2\right)$$

$y = 3f(x)$, $3 \times$ y - coords

$(2,-4)$

(c)

x	<	0	>	<	2	>
$f'(x)$	+	0	-	-	0	+
x-axis	above	on	below	below	on	above

2
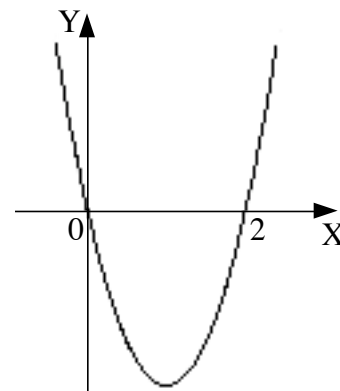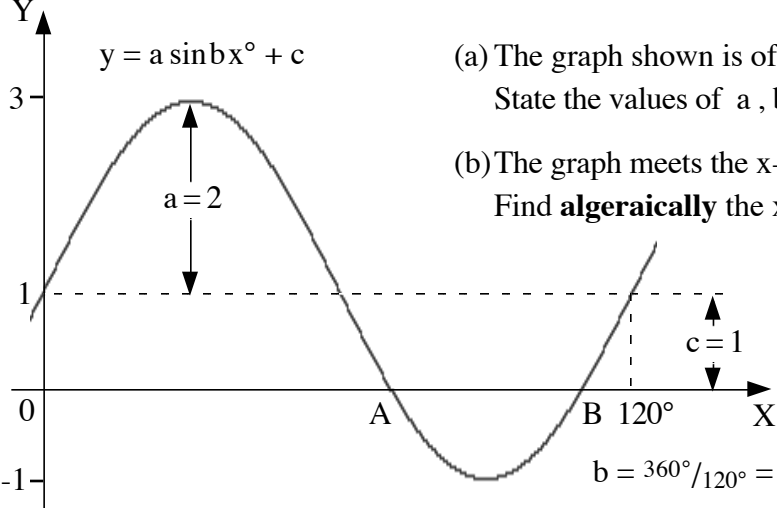
2. Find $\dfrac{d}{dt}\left(\sqrt{t^3}-\dfrac{1}{\sqrt{t}}\right)$, writing your answer in root form.　　　　(4)

$= \dfrac{d}{dt}\left(t^{\frac{3}{2}}-t^{-\frac{1}{2}}\right)$

$= \tfrac{3}{2}t^{\frac{1}{2}}+\tfrac{1}{2}t^{-\frac{3}{2}}$

$= \tfrac{3}{2}\sqrt{t}+\dfrac{1}{2\sqrt{t^3}}$

3. The line passing through the point $(2,-3)$ makes an angle of $135°$ with the positive direction of the x-axis. Find the equation of the line, writing your answer in the form $Ax + By + C = 0$.　　(3)

$m = \tan\theta°$

$= \tan135°$

$= -\tan45°$

$= -1$

$m = -1 \quad \overset{a \quad b}{\left(2,-3\right)}$

$y - b = m(x - a)$

$y-(-3) = -1(x-2)$

$y + 3 = -x + 2$

$x + y + 1 = 0$

4.

$y = a\sin bx° + c$

(a) The graph shown is of the form $y = a\sin bx° + c$. State the values of a, b and c.　　(3)

(b) The graph meets the x-axis at the points A and B. Find **algeraically** the x-coordinates of points A and B.　　(4)

$b = {}^{360°}/_{120°} = 3$

(a) $a = 2$, $b = 3$, $c = 1$

(b)

$2\sin3x° + 1 = 0$

$2\sin3x° = -1$

$\sin3x° = -\dfrac{1}{2}$

$3x = 210 , 330$

$x = 70 , 110$

A, S, T, C is where functions are positive:

sin +	S	A	sin +
180 - a = 150°		a = $\sin^{-1} {}^1/_2 = 30°$	
180 + a = 210°		360 - a = 330°	
sin -	T	C	sin -