

## INDICES RULES:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$\frac{x^{-3} \times x^9}{x^4} = \frac{x^6}{x^4} = x^2$$

$-3 + 9 = 6$   
 $6 - 4 = 2$

$$a^0 = 1$$

$$x^{-3} \times x^3 = x^0 = 1$$

$-3 + 3 = 0$

$$\frac{1}{a^p} = a^{-p}$$

$$\frac{1}{x^3} = x^{-3}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\sqrt[3]{x^4} = x^{\frac{4}{3}}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n$$

$$(2x^{-2})^3 = 2^3 x^{-6} = 8x^{-6}$$

$-2 \times 3 = -6$

Writing in the form  $a^n$

$$\frac{4}{x^3} = 4x^{-3}$$

$$\frac{1}{4x^3} = \frac{1}{4}x^{-3}$$

$$\frac{3}{4x^3} = \frac{3}{4}x^{-3}$$

$$\frac{1}{\sqrt[3]{x^4}} = \frac{1}{x^{\frac{4}{3}}} = x^{-\frac{4}{3}}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$$

$$x + \frac{1}{x} = x + x^{-1}$$

$$\sqrt{x^3} - \frac{1}{\sqrt[3]{x}} = x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}} = x^{\frac{3}{2}} - x^{-\frac{1}{3}}$$

Brackets:

$$x^{-3} (x^3 + x^{-2}) = x^0 + x^{-5} = 1 + x^{-5}$$

*(-3 + 3 = 0)*      *(-3 + -2 = -5)*

$$2x^{-3} (4x^3 + 3x^{-2}) = 8x^0 + 6x^{-5} = 8 + 6x^{-5}$$

$$(1 - x^{-1})(1 - x^{-1}) = 1 - x^{-1} - x^{-1} + x^{-2} = 1 - 2x^{-1} + x^{-2}$$

*(-1 + -1 = -2)*

$$x^{-\frac{1}{2}} (x^{\frac{1}{2}} - x^{-\frac{3}{2}}) = x^0 - x^{-2} = 1 - x^{-2}$$

*(-\frac{1}{2} + \frac{1}{2} = 0)*      *(-\frac{1}{2} + -\frac{3}{2} = -2)*

$$x^{\frac{1}{3}} (x^{\frac{2}{3}} - x^{-\frac{4}{3}}) = x^1 - x^{-1} = x - x^{-1}$$

*(\frac{1}{3} + \frac{2}{3} = 1)*      *(\frac{1}{3} + -\frac{4}{3} = -1)*

Quotients:

$$\frac{(x-3)(x+3)}{2x^3} = \frac{x^2 - 9}{2x^3}$$
$$\frac{x - \sqrt{x}}{\sqrt{x^3}} = \frac{x^1 - x^{\frac{1}{2}}}{x^{\frac{3}{2}}}$$
$$= \frac{x^2}{2x^3} - \frac{9}{2x^3} = \frac{x^1}{x^{\frac{3}{2}}} - \frac{x^{\frac{1}{2}}}{x^{\frac{3}{2}}}$$

$$= \frac{1}{2}x^{-1} - \frac{9}{2}x^{-3}$$

*(2 - 3 = -1)*

$$= x^{-\frac{1}{2}} - x^{-1}$$

*(1 - \frac{3}{2} = -\frac{1}{2})*  
*(\frac{1}{2} - \frac{3}{2} = -1)*

INDICES EXERCISE: write in the form  $a^n$

1. (a)  $4x^3 \times 3x^2$  (b)  $2x^{-5} \times 4x^3$   
 (c)  $2x \times x^3 \times 5x^2$  (d)  $4x^{-1} \times 3x^{-4}$   
 (e)  $3x^{\frac{3}{2}} \times 2x^{\frac{1}{2}}$  (f)  $2x^{\frac{5}{3}} \times 5x^{\frac{2}{3}}$   
 (g)  $x^{-\frac{3}{4}} \times x^{-\frac{1}{4}}$  (h)  $x^{\frac{3}{2}} \times x^{-\frac{1}{4}}$   
 (i)  $\frac{2x^2}{x^5}$  (j)  $\frac{x^3}{2x^7}$  (k)  $\frac{6x^4}{3x^{-1}}$  (l)  $\frac{2x^{-5}}{4x^{-2}}$   
 (m)  $\frac{x^{\frac{5}{4}}}{x^{\frac{1}{2}}}$  (n)  $\frac{x^{\frac{1}{3}}}{x^{-\frac{5}{3}}}$  (o)  $\frac{x^{-\frac{1}{5}}}{x^{-\frac{3}{5}}}$  (p)  $\frac{x^{\frac{1}{2}}}{x^{\frac{3}{4}}}$   
 (q)  $\frac{6x^4 \times 2x^3}{3x^5}$  (r)  $\frac{x^{\frac{7}{2}} \times x^{\frac{1}{2}}}{x^{\frac{3}{2}}}$  (s)  $\frac{x^{-\frac{5}{4}} \times x^{\frac{1}{2}}}{x^{-\frac{1}{4}}}$

2. (a)  $\frac{3}{x^4}$  (b)  $\frac{1}{3x^2}$  (c)  $\sqrt[4]{x^3}$  (d)  $\frac{5}{2x^3}$   
 (e)  $\sqrt[3]{x^5}$  (f)  $x + \frac{2}{x}$  (g)  $\frac{1}{\sqrt[4]{x^5}}$  (h)  $\frac{1}{2x}$   
 (i)  $\frac{1}{\sqrt[5]{x^3}}$  (j)  $\frac{1}{4x^5}$  (k)  $\frac{x^3}{4} + \frac{4}{x^3}$  (l)  $\frac{2}{3x} - 5x$   
 (m)  $\frac{6}{\sqrt[3]{x^2}}$  (n)  $3x^2 - \frac{1}{3x^2}$  (o)  $\frac{1}{\sqrt[5]{x}}$  (p)  $\frac{7}{3x^2}$   
 (q)  $\frac{5}{\sqrt[3]{x}}$  (r)  $\frac{4}{3\sqrt{x}}$  (s)  $\frac{1}{5\sqrt{x}}$  (t)  $\frac{3}{4\sqrt{x^5}}$   
 (u)  $\frac{1}{\sqrt{x}} + \sqrt{x}$  (v)  $\frac{2}{\sqrt{x^3}} - 3\sqrt{x}$

3. (a)  $x^{-2}(x^3 - x^{-2})$  (b)  $x^{-1}(x^5 + x^{-3})$   
 (c)  $3x^2(2x - x^{-3})$  (d)  $2x^4(x^{-2} + 3x^{-5})$   
 (e)  $(x^2 - x^{-2})(x^2 + x^{-2})$  (f)  $(1 - 2x^3)(1 - 3x^2)$   
 (g)  $(3 - x^{-2})(3 - x^2)$  (h)  $(x - x^{-3})(x + x^{-3})$   
 (i)  $x^{\frac{5}{3}}(x^{\frac{1}{3}} - x^{-\frac{2}{3}})$  (j)  $x^{-\frac{1}{4}}(x^{\frac{1}{2}} + x^{-\frac{3}{4}})$   
 (k)  $\sqrt[5]{x}(\sqrt[5]{x^3} - 1)$  (l)  $\sqrt{x^3}(\sqrt{x} - 4x)$   
 (m)  $(x^{\frac{1}{4}} - x^{-\frac{1}{4}})(x^{\frac{1}{4}} - x^{-\frac{1}{4}})$  (n)  $(x^{\frac{2}{3}} - x^{-\frac{4}{3}})(x^{\frac{2}{3}} - x^{-\frac{4}{3}})$   
 (o)  $(\sqrt{x} - 1)(\sqrt{x} + 1)$  (p)  $(\sqrt[3]{x} - 1)^2$

4. (a)  $\frac{x^2 + 5x^4}{x^2}$  (b)  $\frac{4x^{-2} - x^3}{2x}$   
 (c)  $\frac{x^5 - 4}{2x^3}$  (d)  $\frac{x^{-2} + 2x}{3x^2}$   
 (e)  $\frac{(x - 2)(x + 4)}{x^2}$  (f)  $\frac{(x - 3)(x^2 + 3)}{x^3}$   
 (g)  $\frac{x^{\frac{2}{3}} + x^{\frac{1}{3}}}{x^{\frac{4}{3}}}$  (h)  $\frac{x^{-\frac{3}{2}} - x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$   
 (i)  $\frac{x^3 - x}{\sqrt{x}}$  (j)  $\frac{x + \sqrt[3]{x^2}}{\sqrt[3]{x}}$   
 (k)  $\frac{x^2 + x}{\sqrt{x^3}}$  (l)  $\frac{x^3 - \sqrt{x}}{\sqrt{x^3}}$

# INDICES EXERCISE: write in the form $a^n$

1. (a)  $4x^3 \times 3x^2$   $\boxed{12x^5}$  (b)  $2x^{-5} \times 4x^3$   $\boxed{8x^{-2}}$   
 (c)  $2x \times x^3 \times 5x^2$   $\boxed{10x^6}$  (d)  $4x^{-1} \times 3x^{-4}$   $\boxed{12x^{-5}}$   
 (e)  $3x^{\frac{3}{2}} \times 2x^{\frac{1}{2}}$   $\boxed{6x^2}$  (f)  $2x^{\frac{5}{3}} \times 5x^{\frac{2}{3}}$   $\boxed{10x^{\frac{7}{3}}}$   
 (g)  $x^{-\frac{3}{4}} \times x^{-\frac{1}{4}}$   $\boxed{x^{-1}}$  (h)  $x^{\frac{3}{2}} \times x^{-\frac{1}{4}}$   $\boxed{x^{\frac{5}{4}}}$

(i)  $\frac{2x^2}{x^5}$   $\boxed{2x^{-3}}$  (j)  $\frac{x^3}{2x^7}$   $\boxed{\frac{1}{2}x^{-4}}$  (k)  $\frac{6x^4}{3x^{-1}}$   $\boxed{2x^5}$  (l)  $\frac{2x^{-5}}{4x^{-2}}$   $\boxed{\frac{1}{2}x^{-3}}$   
 (m)  $\frac{x^{\frac{5}{4}}}{x^{\frac{1}{2}}}$   $\boxed{x^{\frac{3}{4}}}$  (n)  $\frac{x^{\frac{1}{3}}}{x^{-\frac{5}{3}}}$   $\boxed{x^2}$  (o)  $\frac{x^{-\frac{1}{5}}}{x^{-\frac{3}{5}}}$   $\boxed{x^{\frac{2}{5}}}$  (p)  $\frac{x^{\frac{1}{2}}}{x^{\frac{3}{4}}}$   $\boxed{x^{-\frac{1}{4}}}$

(q)  $\frac{6x^4 \times 2x^3}{3x^5}$   $\boxed{4x^2}$  (r)  $\frac{x^{\frac{7}{2}} \times x^{\frac{1}{2}}}{x^{\frac{3}{2}}}$   $\boxed{x^{\frac{5}{2}}}$  (s)  $\frac{x^{-\frac{5}{4}} \times x^{\frac{1}{2}}}{x^{-\frac{1}{2}}}$   $\boxed{x^{-\frac{1}{4}}}$

2. (a)  $\frac{3}{x^4}$   $\boxed{3x^{-4}}$  (b)  $\frac{1}{3x^2}$   $\boxed{\frac{1}{3}x^{-2}}$  (c)  $\sqrt[4]{x^3}$   $\boxed{x^{\frac{3}{4}}}$  (d)  $\frac{5}{2x^3}$   $\boxed{\frac{5}{2}x^{-3}}$   
 (e)  $\sqrt[3]{x^5}$   $\boxed{x^{\frac{5}{3}}}$  (f)  $x + \frac{2}{x}$   $\boxed{x + 2x^{-1}}$  (g)  $\frac{1}{\sqrt[4]{x^5}}$   $\boxed{x^{-\frac{5}{4}}}$  (h)  $\frac{1}{2x}$   $\boxed{\frac{1}{2}x^{-1}}$   
 (i)  $\frac{1}{\sqrt[5]{x^3}}$   $\boxed{x^{-\frac{3}{5}}}$  (j)  $\frac{1}{4x^5}$   $\boxed{\frac{1}{4}x^{-5}}$  (k)  $\frac{x^3}{4} + \frac{4}{x^3}$   $\boxed{\frac{1}{4}x^3 + 4x^{-3}}$  (l)  $\frac{2}{3x} - 5x$   $\boxed{\frac{2}{3}x^{-1} - 5x}$   
 (m)  $\frac{6}{\sqrt[3]{x^2}}$   $\boxed{6x^{-\frac{2}{3}}}$  (n)  $3x^2 - \frac{1}{3x^2}$   $\boxed{3x^2 + \frac{1}{3}x^{-2}}$  (o)  $\frac{1}{\sqrt[5]{x}}$   $\boxed{x^{-\frac{1}{5}}}$  (p)  $\frac{7}{3x^2}$   $\boxed{\frac{7}{3}x^{-2}}$   
 (q)  $\frac{5}{\sqrt[3]{x}}$   $\boxed{5x^{-\frac{1}{3}}}$  (r)  $\frac{4}{3\sqrt{x}}$   $\boxed{\frac{4}{3}x^{-\frac{1}{2}}}$  (s)  $\frac{1}{5\sqrt{x}}$   $\boxed{\frac{1}{5}x^{-\frac{1}{2}}}$  (t)  $\frac{3}{4\sqrt{x^5}}$   $\boxed{\frac{3}{4}x^{-\frac{5}{2}}}$   
 (u)  $\frac{1}{\sqrt{x}} + \sqrt{x}$   $\boxed{x^{-\frac{1}{2}} + x^{\frac{1}{2}}}$  (v)  $\frac{2}{\sqrt{x^3}} - 3\sqrt{x}$   $\boxed{2x^{-\frac{3}{2}} - 3x^{\frac{1}{2}}}$

3. (a)  $x^{-2}(x^3 - x^{-2})$  (b)  $x^{-1}(x^5 + x^{-3})$   
 $x - x^{-4}$   $x^4 + x^{-4}$   
(c)  $3x^2(2x - x^{-3})$  (d)  $2x^4(x^{-2} + 3x^{-5})$   
 $6x^3 - 3x^{-1}$   $2x^2 + 6x^{-1}$   
(e)  $(x^2 - x^{-2})(x^2 + x^{-2})$  (f)  $(1 - 2x^3)(1 - 3x^2)$   
 $x^4 - x^{-4}$   $1 - 2x^3 - 3x^2 + 6x^5$   
(g)  $(3 - x^{-2})(3 - x^2)$  (h)  $(x - x^{-3})(x + x^{-3})$   
 $10 - 3x^2 - 3x^{-2}$   $x^2 - x^{-6}$   
(i)  $x^{\frac{5}{3}}(x^{\frac{1}{3}} - x^{-\frac{2}{3}})$  (j)  $x^{-\frac{1}{4}}(x^{\frac{1}{2}} + x^{-\frac{3}{4}})$   
 $x^2 - x$   $x^{\frac{1}{4}} + x^{-1}$   
(k)  $\sqrt[5]{x}(\sqrt[5]{x^3} - 1)$  (l)  $\sqrt{x^3}(\sqrt{x} - 4x)$   
 $x^{\frac{4}{5}} - x^{\frac{1}{5}}$   $x^2 - 4x^{\frac{5}{2}}$   
(m)  $(x^{\frac{1}{4}} - x^{-\frac{1}{4}})(x^{\frac{1}{4}} - x^{-\frac{1}{4}})$  (n)  $(x^{\frac{2}{3}} - x^{-\frac{4}{3}})(x^{\frac{2}{3}} - x^{-\frac{4}{3}})$   
 $x^{\frac{1}{2}} - 2 + x^{-\frac{1}{2}}$   $x^{\frac{4}{3}} - 2x^{-\frac{2}{3}} + x^{-\frac{8}{3}}$   
(o)  $(\sqrt{x} - 1)(\sqrt{x} + 1)$   $x - 1$  (p)  $(\sqrt[3]{x} - 1)^2 x^{\frac{2}{3}} - 2x^{\frac{1}{3}} + 1$

4. (a)  $\frac{x^2 + 5x^4}{x^2}$   $1 + 5x^2$  (b)  $\frac{4x^{-2} - x^3}{2x}$   $2x^{-3} - \frac{1}{2}x^2$   
(c)  $\frac{x^5 - 4}{2x^3}$   $\frac{1}{2}x^2 - 2x^{-3}$  (d)  $\frac{x^{-2} + 2x}{3x^2}$   $\frac{1}{3}x^{-4} + \frac{2}{3}x^{-1}$   
(e)  $\frac{(x-2)(x+4)}{x^2}$   $1 + 2x^{-1} - 8x^{-2}$  (f)  $\frac{(x-3)(x^2+3)}{x^3}$   
 $1 - 3x^{-1} + 3x^{-2} - 9x^{-3}$   
(g)  $\frac{x^{\frac{2}{3}} + x^{\frac{1}{3}}}{x^{\frac{4}{3}}}$   $x^{-\frac{2}{3}} + x^{-1}$  (h)  $\frac{x^{-\frac{3}{2}} - x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$   $x^{-2} - 1$   
(i)  $\frac{x^3 - x}{\sqrt{x}}$   $x^{\frac{5}{2}} - x^{\frac{1}{2}}$  (j)  $\frac{x + \sqrt[3]{x^2}}{\sqrt[3]{x}}$   $x^{\frac{2}{3}} + x^{\frac{1}{3}}$   
(k)  $\frac{x^2 + x}{\sqrt{x^3}}$   $x^{\frac{1}{2}} + x^{-\frac{1}{2}}$  (l)  $\frac{x^3 - \sqrt{x}}{\sqrt{x^3}}$   $x^{\frac{3}{2}} - x^{-1}$