WAVE FUNCTION

Functions of the form $y = a\cos x + b\sin x$

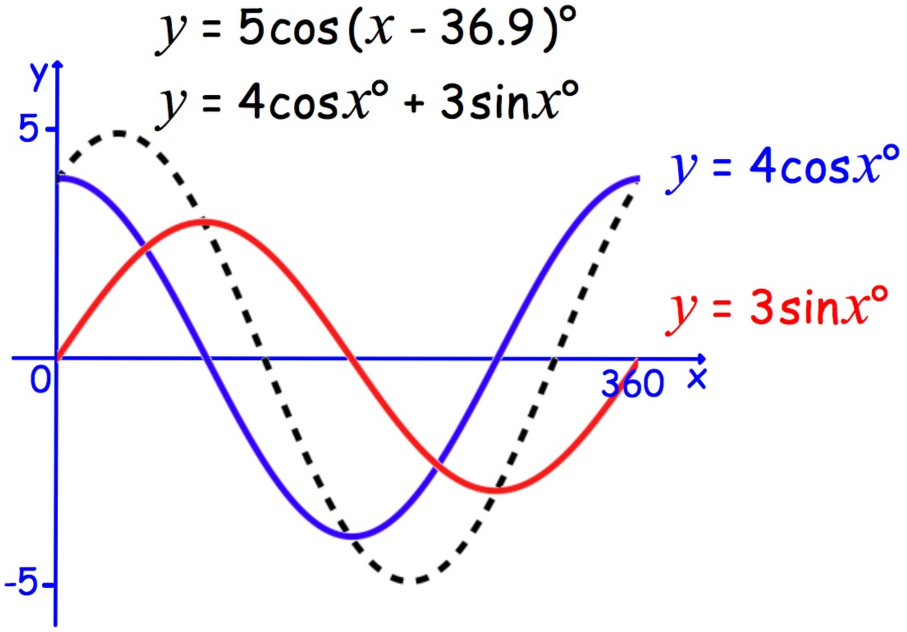
A sum of two functions,
the resultant wave has: increased amplitude, R
change of phase, a

so can be written in forms: $R\cos(x \pm a)$ or $R\sin(x \pm a)$

EXPANSION FORMULAE are used.

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$



(1) Write $4\cos x^\circ + 3\sin x^\circ$ in the form $R\cos(x - a)^\circ$

use expansion formulae:

$$4\cos x^\circ + 3\sin x^\circ = R\cos(x - a)^\circ$$
$$= R\cos x^\circ \cos a^\circ + R\sin x^\circ \sin a^\circ$$
$$4\cos x^\circ + 3\sin x^\circ = (R\cos a^\circ)\cos x^\circ + (R\sin a^\circ)\sin x^\circ$$

comparing sides: $R\sin a^\circ = 3$

$R\cos a^\circ = 4$

Solve for R and a using Trig. identities:

squaring $R^2\sin^2 a^\circ = 9$

$R^2\cos^2 a^\circ = 16$

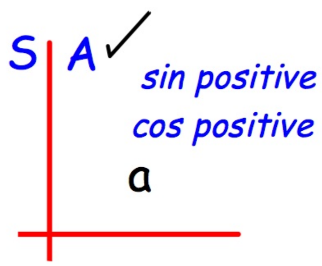
adding

$$R^2(\sin^2 a^\circ + \cos^2 a^\circ) = 25$$
$$R^2 \times 1 = 25$$
$$R = 5$$

$$\frac{R\sin a^\circ}{R\cos a^\circ} = \frac{+3}{+4}$$

$\tan a^\circ = {}^3/_4$

$a = 36.8698....$



only one quadrant will satisfy the signs of both $R\sin a^\circ$ and $R\cos a^\circ$

$$4\cos x^\circ + 3\sin x^\circ = \underline{\underline{5\cos(x - 36.9)^\circ}}$$

(2) Write $\cos x^\circ - \sqrt{3}\sin x^\circ$ in the form $R\sin(x - a)^\circ$

$$\cos x^\circ - \sqrt{3}\sin x^\circ = R\sin(x - a)^\circ$$

$$= R\sin x^\circ \cos a^\circ - R\cos x^\circ \sin a^\circ$$

$$-\sqrt{3}\sin x^\circ - (-1)\cos x^\circ = (R\cos a^\circ)\sin x^\circ - (R\sin a^\circ)\cos x^\circ$$

$$R\sin a^\circ = -1$$

$$R\cos a^\circ = -\sqrt{3}$$

$$R^2 = (-1)^2 + (-\sqrt{3})^2$$

$$= 1 + 3$$

$$= 4$$

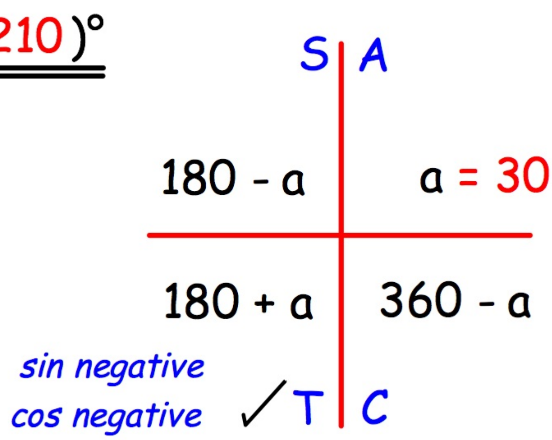
$$R = 2$$

$$\frac{R\sin a^\circ}{R\cos a^\circ} = \frac{-1}{-\sqrt{3}}$$

$$\tan a^\circ = \frac{1}{\sqrt{3}}$$

$$a = 210$$

$$\cos x^\circ - \sqrt{3}\sin x^\circ = \underline{\underline{2\sin(x - 210)^\circ}}$$



(3) Write $\cos 2x° - \sin 2x°$ in the form $R\sin(2x + a)°$

NOTE: do not use expansions for sin2A or cos2A

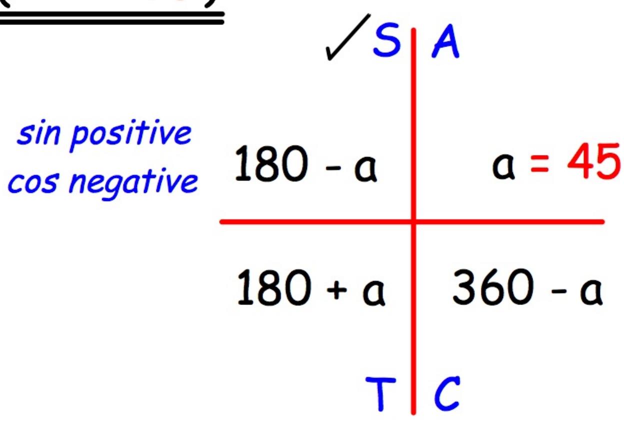
$$\cos 2x° - \sin 2x° = R\sin(2x + a)°$$
$$= R\sin 2x° \cos a° + R\cos 2x° \sin a°$$
$$(-1)\sin 2x° + 1\cos 2x° = (R\cos a°)\sin 2x° + (R\sin a°)\cos 2x°$$

$$R\sin a° = +1$$
$$R\cos a° = -1$$

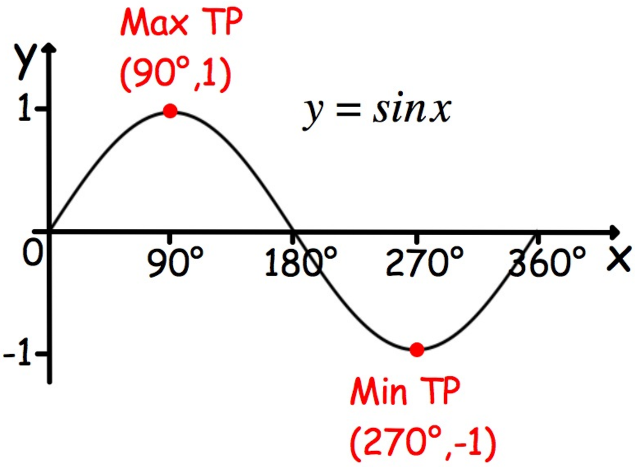
$$R^2 = 1^2 + (-1)^2$$
$$= 1 + 1$$
$$= 2$$
$$R = \sqrt{2}$$

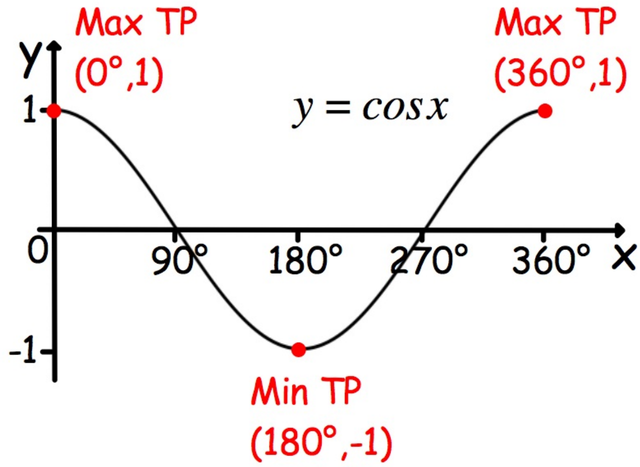
$$\frac{R\sin a°}{R\cos a°} = \frac{+1}{-1}$$
$$\tan a° = -1$$
$$a = 135$$

$$\cos 2x° - \sin 2x° = \underline{\underline{\sqrt{2}\sin(2x - 135)°}}$$



MAXIMUM and MINIMUM VALUES



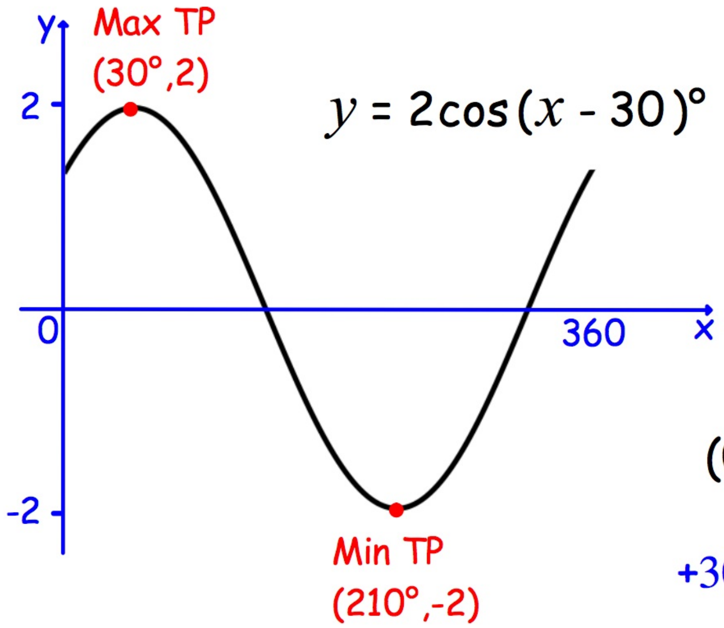


TRANSFORMATIONS: $R\cos(x \pm a)$

stretch R units vertically (R)

$-a$ shift a° RIGHT

$+a$ shift a° LEFT



(0°, 1) → +30°, ×2 → (30°, 2) MAX. TP

(180°, -1) → +30°, ×2 → (210°, -2) MIN. TP

NOTE: these are STATIONARY POINTS

(1) $5\sin(2x - 30)^\circ + 3 \quad , \quad 0 \leq x \leq 180$

MAXIMUM

$5\sin 90^\circ + 3$
$= 5 \times 1 + 3$
$= 8$

$2x - 30 = 90$
$2x = 120$
$x = 60$

MINIMUM

$5\sin 270^\circ + 3$
$= 5 \times (-1) + 3$
$= -2$

$2x - 30 = 270$
$2x = 300$
$x = 150$

MAX (60 , 8) and MIN (150 , -2)

(2) $5\cos(2x - 30)^\circ + 3 \quad , \quad 0 \leq x \leq 180$

MAXIMUM

$5\cos 0^\circ + 3$
$= 5 \times 1 + 3$
$= 8$

$2x - 30 = 0$
$2x = 30$
$x = 15$

MINIMUM

$5\cos 180^\circ + 3$
$= 5 \times (-1) + 3$
$= -2$

$2x - 30 = 180$
$2x = 210$
$x = 105$

MAX (15 , 8) and MIN (105 , -2)

EQUATIONS

Equations of the form $a\cos x + b\sin x = c$

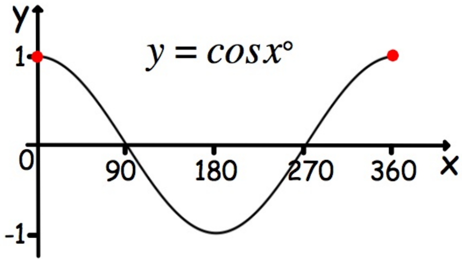
Express in the form $R\cos(x - a) = c$

or similar

(1)

$$4\cos x^\circ + 3\sin x^\circ = 5$$

$$5\cos(x - 36.9)^\circ = 5$$



$$\cos(x - 36.9)^\circ = 1$$

$$x - 36.9 = 0$$

$$\underline{\underline{x = 36.9}}$$

(2)

$$\cos x^\circ - \sqrt{3}\sin x^\circ = \sqrt{3}$$

$$2\sin(x - 210)^\circ = \sqrt{3}$$

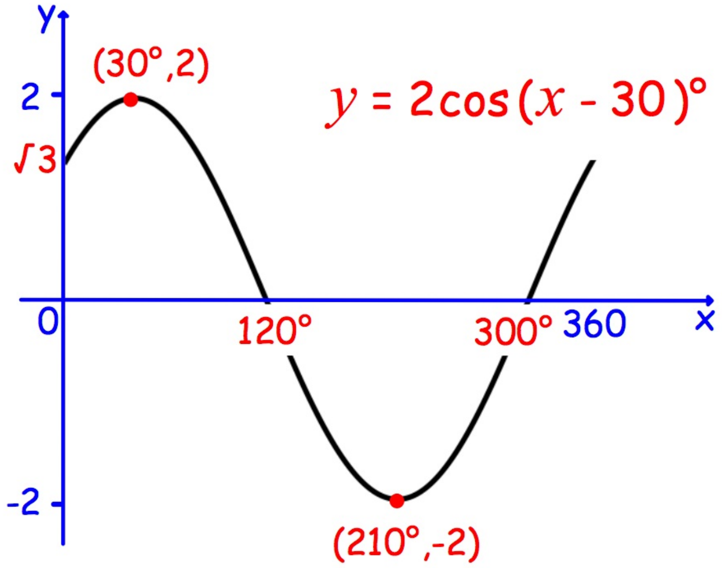
$$\sin(x - 210)^\circ = \sqrt{3}/2$$

$$x - 210 = 60\ ,\ 120$$

$$\underline{\underline{x = 270\ ,\ 330}}$$

sin positive ✓ S	A ✓ sin positive
180 - a	$a = \sin^{-1}(\sqrt{3}/2) = 60$
180 + a	360 - a
T	C

SKETCH $y = \sqrt{3}\cos x^\circ + \sin x^\circ$



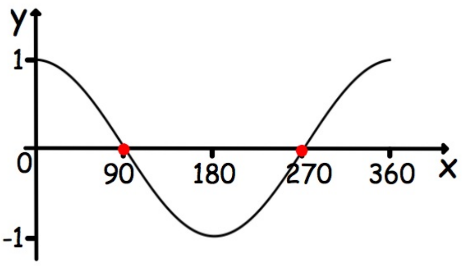
y-axis $x = 0$

$y = \sqrt{3}\cos x^\circ + \sin x^\circ$ or $y = 2\cos(x - 30)^\circ$

$y = \sqrt{3}\cos 0^\circ + \sin 0^\circ$
$= \sqrt{3} \times 1 + 0$
$= \sqrt{3}$

$y = 2\cos(0 - 30)^\circ$
$= 2\cos(-30)^\circ$
$= 2 \times \sqrt{3}/2$
$= \sqrt{3}$

x-axis $y = 0$



$2\cos(x - 30)^\circ = 0$
$\cos(x - 30)^\circ = 0$
$x - 30 = 90$ or 270
$x = 120$ or 300