P73 Ex2

(4) $f(x) = 3x - x^3$

$f'(x) = 3 - 3x^2$

$\quad = 3(1 - x^2)$

$\quad = 3(1 + x)(1 - x)$

x		$\to$ -1 $\to$			$\to$ 1 $\to$	
1 + x	-	0	+	+	+	+
1 - x	+	+	+	+	0	-
f'(x)	-	0	+	+	0	-
shape	↘	_	↗	↗	‾	↘
nature		min. TP			max. TP	
		(-1,-2)			(1,2)	

stationary $f'(x) = 0$

$3(1 + x)(1 - x) = 0$

$x = -1$ or $x = 1$

$f(-1) = 3 \times (-1) - (-1)^3 = -2$

$f(1) = 3 \times 1 - 1^3 = 2$
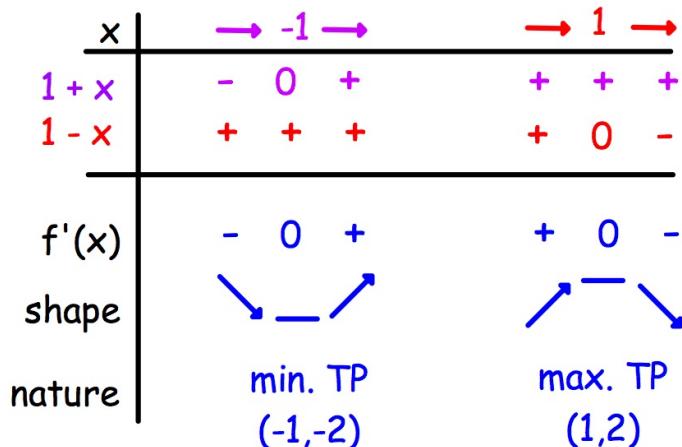
increasing $f'(x) > 0$ $-1 < x < 1$

decreasing $f'(x) < 0$ $x < -1$ and $x > 1$

(5) $f(x) = 3x^4 - 4x^3$

$f'(x) = 12x^3 - 12x^2$

$\quad = 12x^2(x - 1)$

x		$\to$ 0 $\to$			$\to$ 1 $\to$	
x^2	+	0	+	+	+	+
x - 1	-	-	-	-	0	+
f'(x)	-	0	-	-	0	+
shape	↘	_	↘	↘	_	↗
nature	point of inflexion			min. TP		
	(0,0)			(1,-1)		

stationary $f'(x) = 0$

$12x^2(x - 1) = 0$

$x = 0$ or $x = 1$

$f(0) = 3 \times 0^4 - 4 \times 0^3 = 0$

$f(1) = 3 \times 1^4 - 4 \times 1^3 = -1$
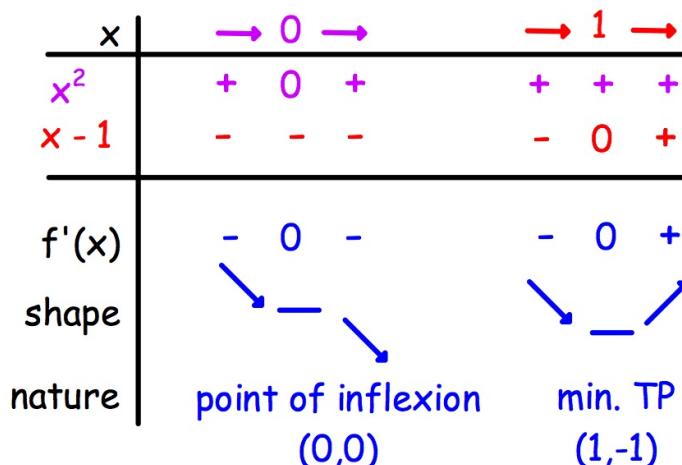
increasing $f'(x) > 0$ $x > 1$

decreasing $f'(x) < 0$ $x < 0$ and $0 < x < 1$

(6) $f(x) = x^4 - 2x^2 + 5$

$f'(x) = 4x^3 - 4x$

$\quad\quad = 4x(x^2 - 1)$

$\quad\quad = 4x(x + 1)(x - 1)$

stationary $f'(x) = 0$

$4x(x + 1)(x - 1) = 0$

$x = -1$ or $x = 0$ or $x = 1$

$f(-1) = (-1)^4 - 2(-1)^2 + 5 = 4$

$f(0) = 0^4 - 2 \times 0^2 + 5 = 5$

$f(1) = 1^4 - 2 \times 1^2 + 5 = 4$

x	→ -1 →			→ 0 →			→ 1 →		
x	−	−	−	−	0	+	+	+	+
x + 1	−	0	+	+	+	+	+	+	+
x − 1	−	−	−	−	−	−	−	0	+
f'(x)	−	0	+	+	0	−	−	0	+
shape	↘ _ ↗			↗ ‾ ↘			↘ _ ↗		
nature	min. TP (-1,4)			max. TP (0,5)			min. TP (1,4)		

increasing $f'(x) > 0$ $-1 < x < 0$ and $x > 1$

decreasing $f'(x) < 0$ $x < -1$ and $0 < x < 1$

P75 Ex3

(8) $f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$
$= 6x(1 - x)$

stationary $f'(x) = 0$
$6x(1 - x) = 0$
$x = 0$ or $x = 1$

$f(0) = 3 \times 0^2 - 2 \times 0^3 = 0$
$f(1) = 3 \times 1^2 - 2 \times 1^3 = 1$

x		→ 0 →			→ 1 →	
6x	-	0	+	+	+	+
1 - x	+	+	+	+	0	-
f'(x)	-	0	+	+	0	-
shape	↘	—	↗	↗	—	↘
nature		min. TP (0,0)			max. TP (1,1)	
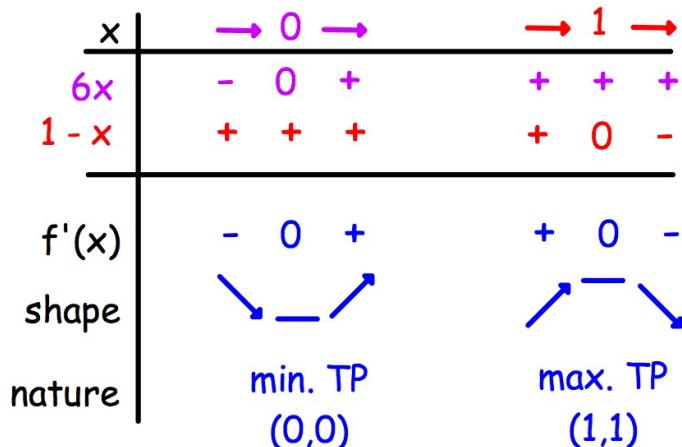
AXES: $y = x^2(3 - 2x)$
$x = 0$ $y = 0^2(3 - 2 \times 0) = 0$ $(0,0)$
$y = 0$ $x^2(3 - 2x) = 0$
    $x = 0$ or $x = \frac{3}{2}$ $(0,0)$, $(\frac{3}{2},0)$
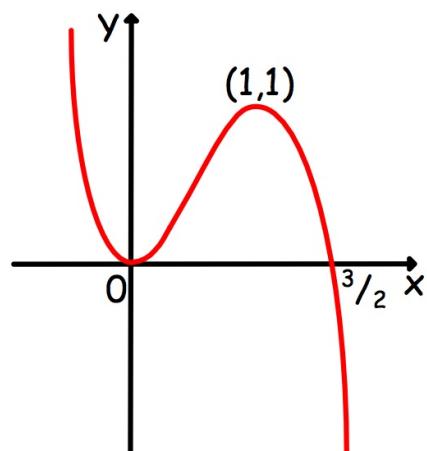
|		
$x \longrightarrow \infty$	$y \longrightarrow -x^3$	
$x \longrightarrow -\infty$	$y \longrightarrow +\infty$	
$x \longrightarrow +\infty$	$y \longrightarrow -\infty$	

(9) $f(x) = x^3 - 6x^2 + 9x$

$f'(x) = 3x^2 - 12x + 9$
$= 3(x^2 - 4x + 3)$
$= 3(x - 1)(x - 3)$

stationary $f'(x) = 0$
$3(x - 1)(x - 3) = 0$
$x = 1$ or $x = 3$

$f(1) = 1(1 - 3)^2 = (-2)^2 = 4$
$f(3) = 3(3 - 3)^2 = 0$

x		→ 1 →			→ 3 →	
x - 1	-	0	+	+	+	+
x - 3	-	-	-	-	0	+
f'(x)	+	0	-	-	0	+
shape	↗	—	↘	↘	—	↗
nature		max. TP (1,4)			min. TP (3,0)	
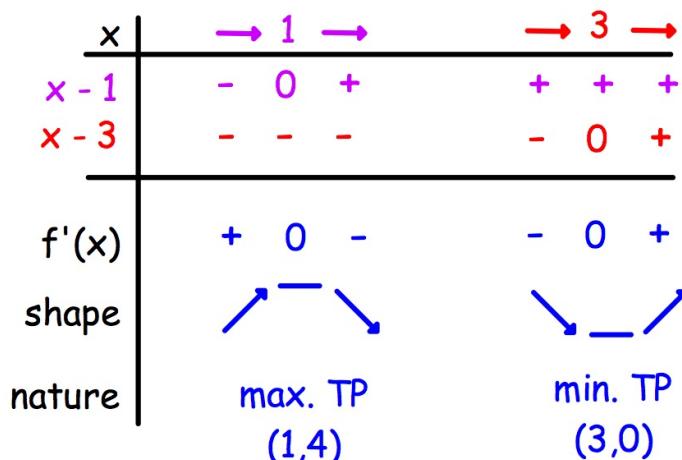
AXES: $y = x(x - 3)^2$
$x = 0$ $y = 0(0 - 3)^2 = 0$ $(0,0)$
$y = 0$ $x(x - 3)^2 = 0$
    $x = 0$ or $x = 3$ $(0,0)$, $(3,0)$
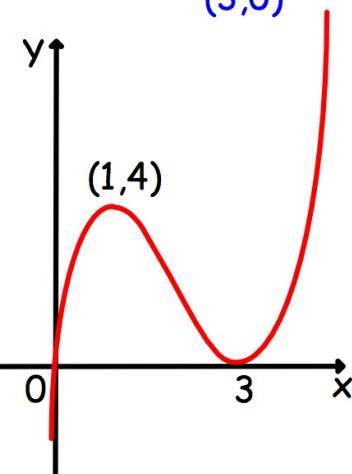
$x \longrightarrow \infty$	$y \longrightarrow +x^3$	
$x \longrightarrow -\infty$	$y \longrightarrow -\infty$	
$x \longrightarrow +\infty$	$y \longrightarrow +\infty$	

(11) $f(x) = 4x^3 - x^4$

$f'(x) = 12x^2 - 4x^3$

$\quad\quad = 4x^2(3 - x)$

stationary $f'(x) = 0$

$\quad 4x^2(3 - x) = 0$

$\quad\quad x = 0 \;\; or \;\; x = 3$

$f(0) = 4 \times 0^3 - 0^4 = 0$

$f(3) = 4 \times 3^3 - 3^4 = 27$

x		$\to 0 \to$			$\to 3 \to$	
x^2	+	0	+	+	+	+
$3 - x$	+	+	+	+	0	−
$f'(x)$	+	0	+	+	0	−
shape						
nature	point of inflexion (0,0)			max. TP (3,27)		
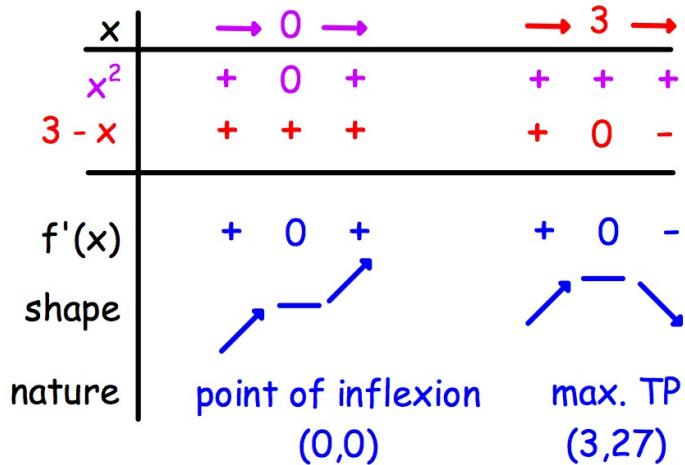
AXES: $y = x^3(4 - x)$

$x = 0 \quad\quad y = 0^3(4 - 0) = 0 \quad\quad (0,0)$

$y = 0 \quad\quad x^3(4 - x) = 0$

$\quad\quad x = 0 \;\; or \;\; x = 4 \quad\quad (0,0), (4,0)$
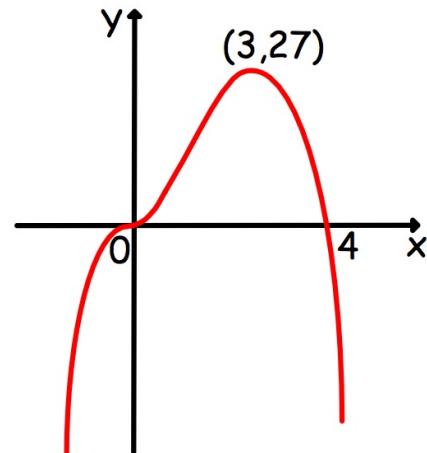
$x \longrightarrow \infty$	$y \longrightarrow -x^4$	
$x \longrightarrow -\infty$	$y \longrightarrow -\infty$	
$x \longrightarrow +\infty$	$y \longrightarrow -\infty$	

(12) $f(x) = 4x^3 + 6x^2$

$f'(x) = 12x^2 + 12x$

$\quad\quad = 12x(x + 1)$

stationary $f'(x) = 0$

$\quad 12x(x + 1) = 0$

$\quad\quad x = 0 \;\; or \;\; x = -1$

$f(0) = 4 \times 0^3 + 6 \times 0^2 = 0$

$f(-1) = 4 \times (-1)^3 + 6 \times (-1)^2 = 2$

x		$\to -1 \to$			$\to 0 \to$	
x	−	−	−	−	0	+
$x + 1$	−	0	+	+	+	+
$f'(x)$	+	0	−	−	0	+
shape						
nature	max. TP (-1,2)			min. TP (0,0)		
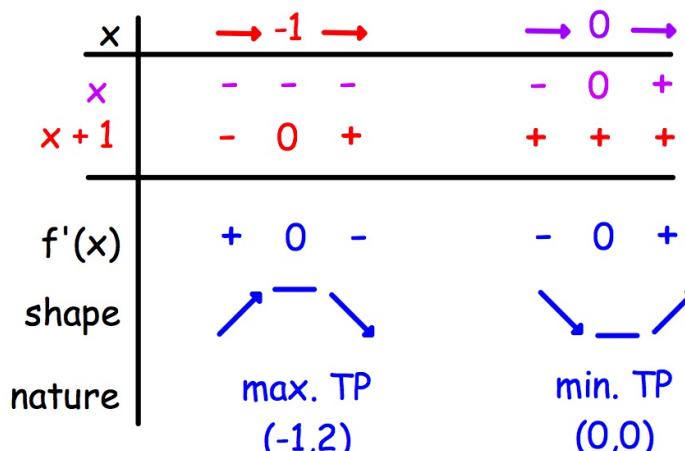
AXES: $y = 2x^2(2x + 3)$

$x = 0 \quad\quad y = 4 \times 0^3 + 6 \times 0^2 = 0 \quad\quad (0,0)$

$y = 0 \quad\quad 2x^2(2x + 3) = 0$

$\quad\quad x = 0 \;\; or \;\; x = -^3/_2 \quad (0,0), (-^3/_2, 0)$

$x \longrightarrow \infty$	$y \longrightarrow +x^3$	
$x \longrightarrow -\infty$	$y \longrightarrow -\infty$	
$x \longrightarrow +\infty$	$y \longrightarrow +\infty$	