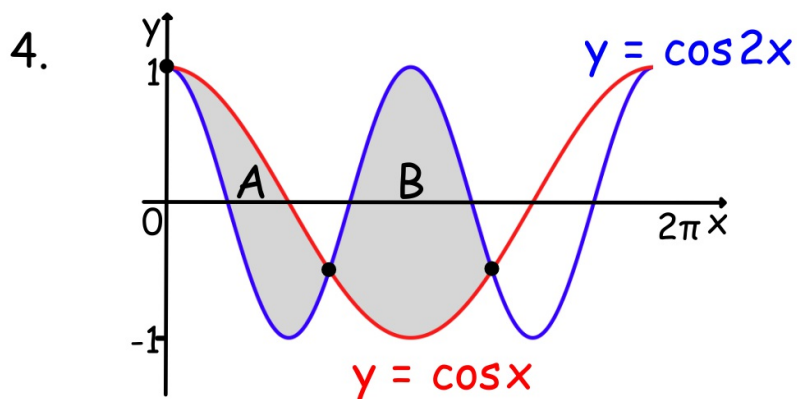
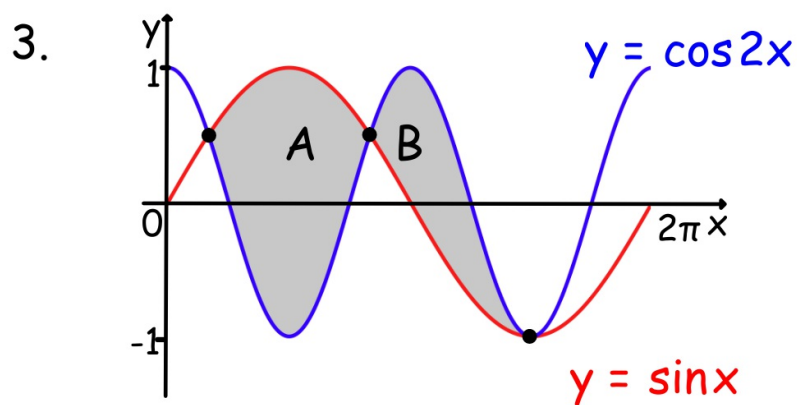
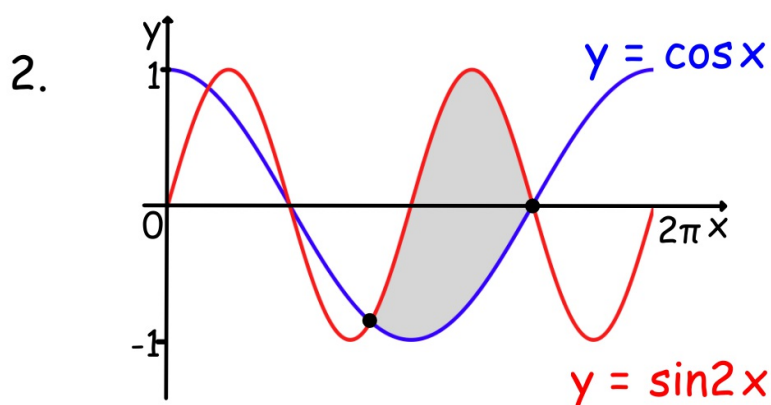
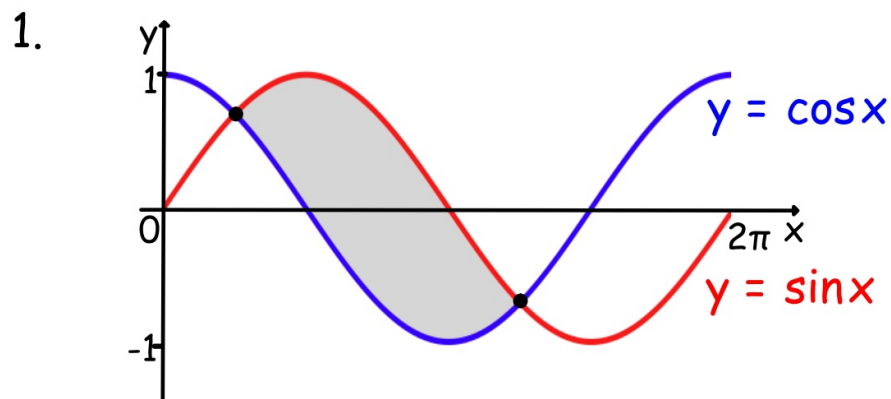
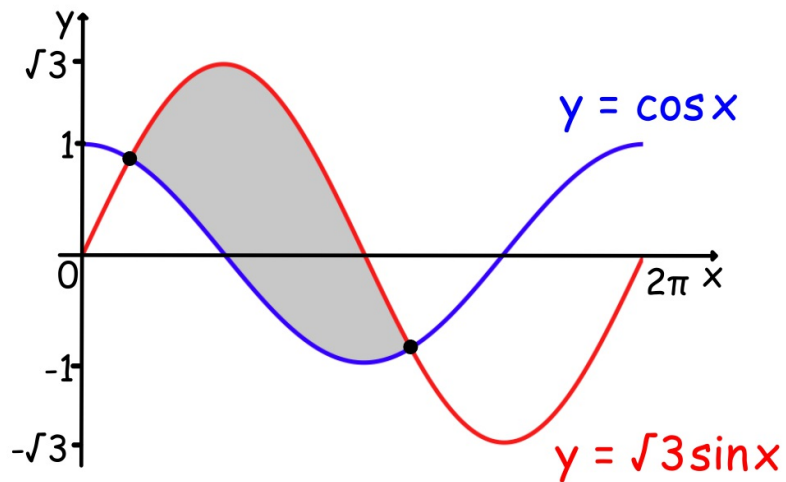


Calculate the shaded areas.



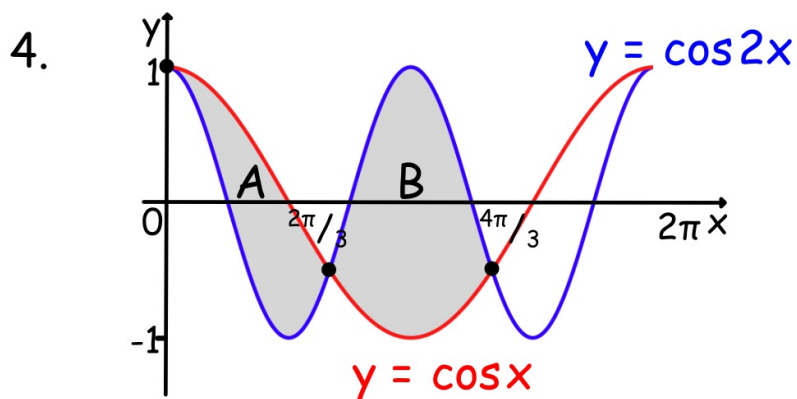
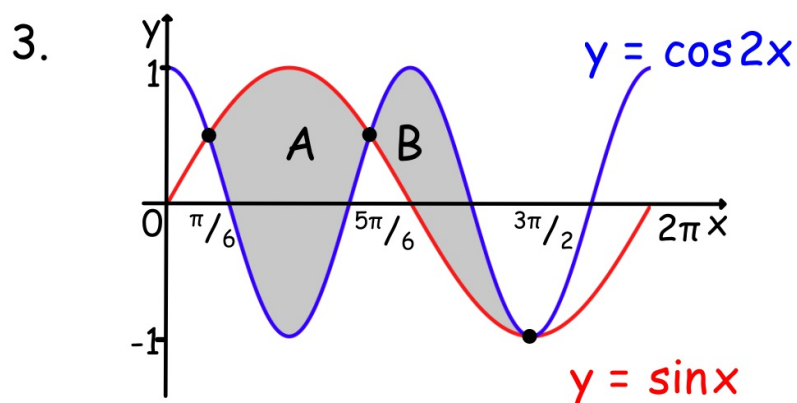
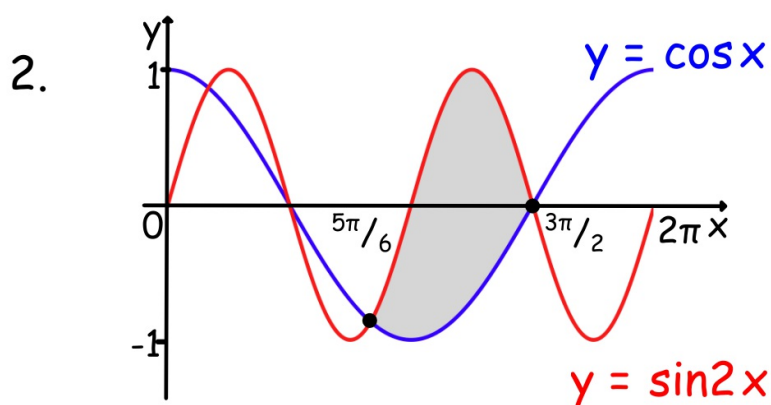
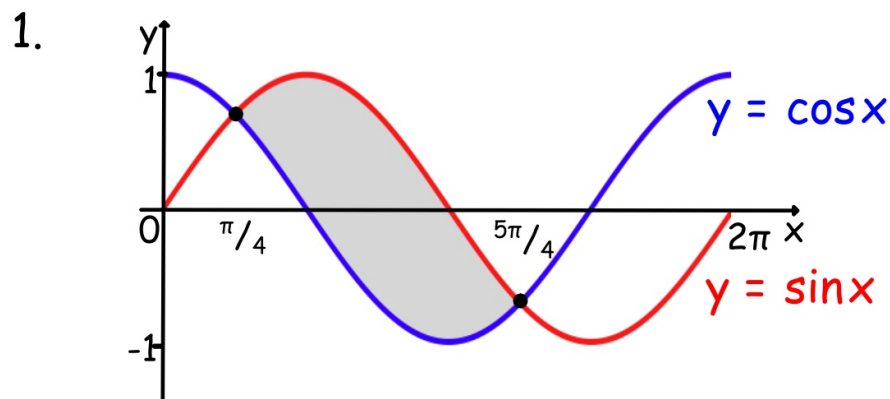
5.



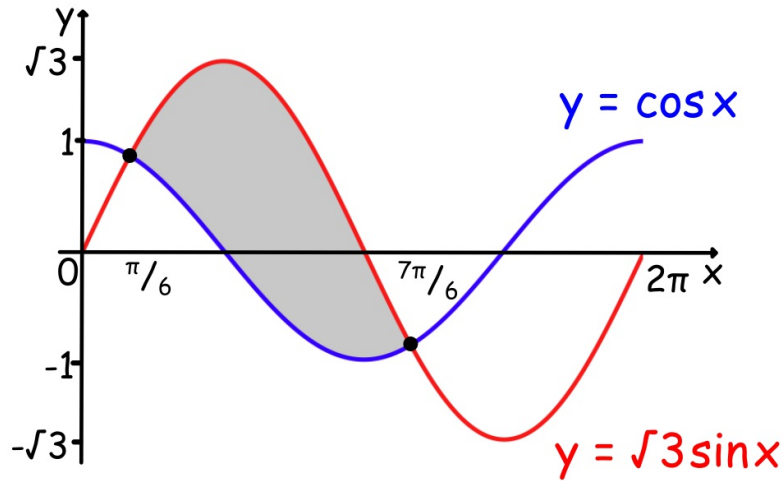
## ANSWERS

1. limits  $\pi/4$  ,  $5\pi/4$   $2\sqrt{2}$  units<sup>2</sup>
2. limits  $5\pi/6$  ,  $3\pi/2$   $9/4$  units<sup>2</sup>
3. limits  $\pi/6$  ,  $5\pi/6$   $3\sqrt{3}/2$  units<sup>2</sup>  
 limits  $5\pi/6$  ,  $3\pi/2$   $3\sqrt{3}/4$  units<sup>2</sup>
4. limits  $0$  ,  $2\pi/3$   $3\sqrt{3}/4$  units<sup>2</sup>  
 limits  $2\pi/3$  ,  $4\pi/3$   $3\sqrt{3}/2$  units<sup>2</sup>
5. limits  $\pi/6$  ,  $7\pi/6$   $4$  units<sup>2</sup>

Calculate the shaded areas.

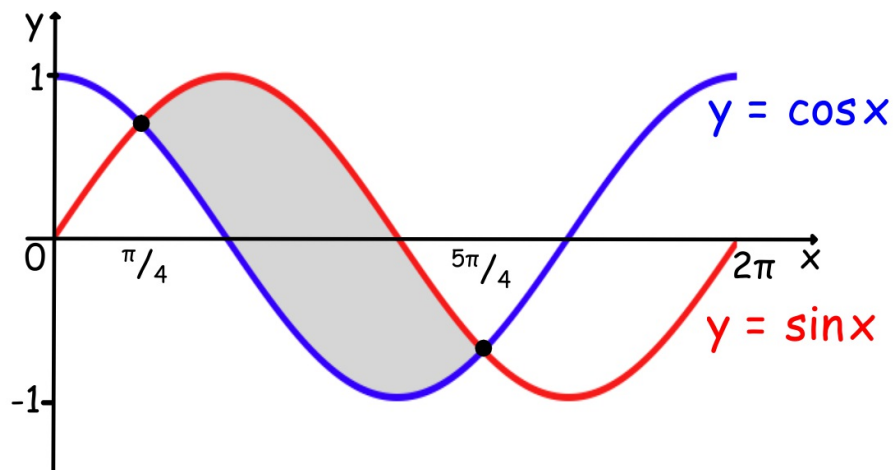


5.



### ANSWERS

1. limits  $\pi/4$  ,  $5\pi/4$   $2\sqrt{2}$  units<sup>2</sup>
2. limits  $5\pi/6$  ,  $3\pi/2$   $9/4$  units<sup>2</sup>
3. limits  $\pi/6$  ,  $5\pi/6$   $3\sqrt{3}/2$  units<sup>2</sup>  
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 limits  $2\pi/3$  ,  $4\pi/3$   $3\sqrt{3}/2$  units<sup>2</sup>
5. limits  $\pi/6$  ,  $7\pi/6$   $4$  units<sup>2</sup>



$$\begin{aligned} \sin x &= \cos x \\ \frac{\sin x}{\cos x} &= 1 \\ \tan x &= 1 \\ x &= \frac{\pi}{4}, \frac{5\pi}{4} \end{aligned}$$

$$\int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx$$

$$= \left[ -\cos x - \sin x \right]_{\pi/4}^{5\pi/4}$$

$$= (-\cos^{5\pi/4} - \sin^{5\pi/4}) - (-\cos^{\pi/4} - \sin^{\pi/4})$$

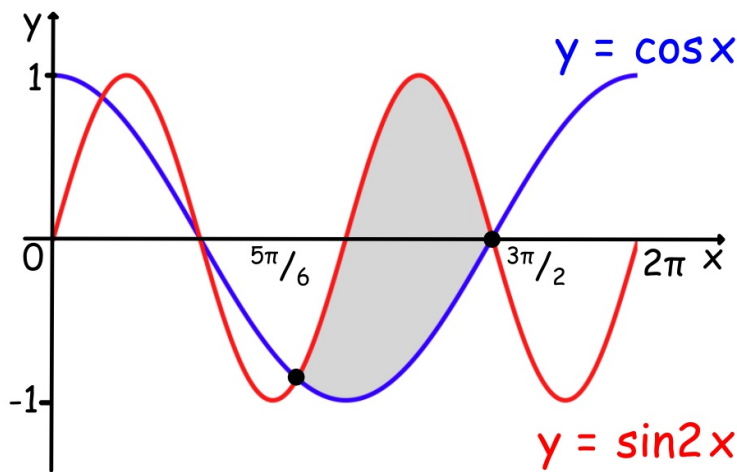
$$= (-(-1/\sqrt{2}) - (-1/\sqrt{2})) - (-1/\sqrt{2} - 1/\sqrt{2})$$

$$= \frac{2}{\sqrt{2}} - \left( \frac{-2}{\sqrt{2}} \right)$$

$$= \sqrt{2} + \sqrt{2}$$

$$= 2\sqrt{2}$$

AREA  $2\sqrt{2}$  units<sup>2</sup>



$$\sin 2x = \cos x$$

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

$$\cos x = 0 \quad \text{or} \quad \sin x = 1/2$$

$$x = \pi/2, 3\pi/2$$

$$\text{or} \quad x = \pi/6, 5\pi/6$$

$$\int_{5\pi/6}^{3\pi/2} (\sin 2x - \cos x) dx$$

$$= \left[ -\frac{1}{2} \cos 2x - \sin x \right]_{5\pi/6}^{3\pi/2}$$

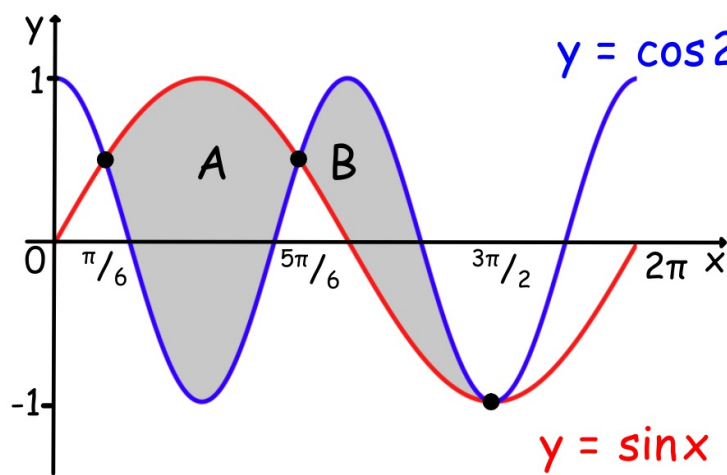
$$= \left( -\frac{1}{2} \cos 3\pi - \sin \frac{3\pi}{2} \right) - \left( -\frac{1}{2} \cos \frac{5\pi}{3} - \sin \frac{5\pi}{6} \right)$$

$$= \left( -\frac{1}{2} (-1) - (-1) \right) - \left( -\frac{1}{2} \times \frac{1}{2} - \frac{1}{2} \right)$$

$$= \frac{3}{2} - \left( -\frac{3}{4} \right)$$

$$= \frac{9}{4}$$

$$\text{AREA } \frac{9}{4} \text{ units}^2$$



$$\sin x = \cos 2x$$

$$\sin x = 1 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = 1/2 \text{ or } \sin x = -1$$

$$x = \pi/6, 5\pi/6,$$

$$\text{or } x = 3\pi/2$$

$$\int_{\pi/6}^{5\pi/6} (\sin x - \cos 2x) dx$$

$$= \left[ -\cos x - \frac{1}{2} \sin 2x \right]_{\pi/6}^{5\pi/6}$$

$$= (-\cos 5\pi/6 - \frac{1}{2} \sin 5\pi/3) - (-\cos \pi/6 - \frac{1}{2} \sin \pi/3)$$

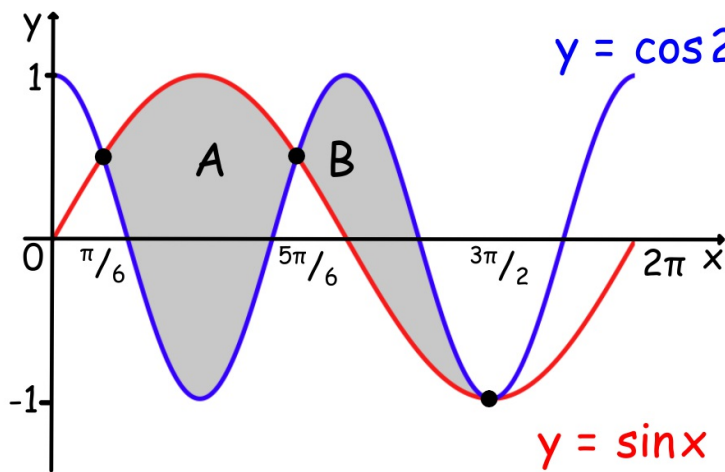
$$= (-(-\sqrt{3}/2) - \frac{1}{2}(-\sqrt{3}/2)) - (-\sqrt{3}/2 - \frac{1}{2} \times \sqrt{3}/2)$$

$$= (\sqrt{3}/2 + \sqrt{3}/4) - (-\sqrt{3}/2 - \sqrt{3}/4)$$

$$= \frac{3\sqrt{3}}{4} + \frac{3\sqrt{3}}{4}$$

$$= \frac{3\sqrt{3}}{2}$$

$$\text{AREA } \frac{3\sqrt{3}}{2} \text{ units}^2$$



$$\sin x = \cos 2x$$

$$\sin x = 1 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = 1/2 \text{ or } \sin x = -1$$

$$x = \pi/6, 5\pi/6,$$

$$\text{or } x = 3\pi/2$$

$$\int_{5\pi/6}^{3\pi/2} (\cos 2x - \sin x) dx$$

$$= \left[ \frac{1}{2} \sin 2x + \cos x \right]_{5\pi/6}^{3\pi/2}$$

$$= \left( \frac{1}{2} \sin 3\pi + \cos \frac{3\pi}{2} \right) - \left( \frac{1}{2} \sin \frac{5\pi}{3} + \cos \frac{5\pi}{6} \right)$$

$$= \left( \frac{1}{2} \times 0 + 0 \right) - \left( \frac{1}{2} \times \left( -\frac{\sqrt{3}}{2} \right) + \left( -\frac{\sqrt{3}}{2} \right) \right)$$

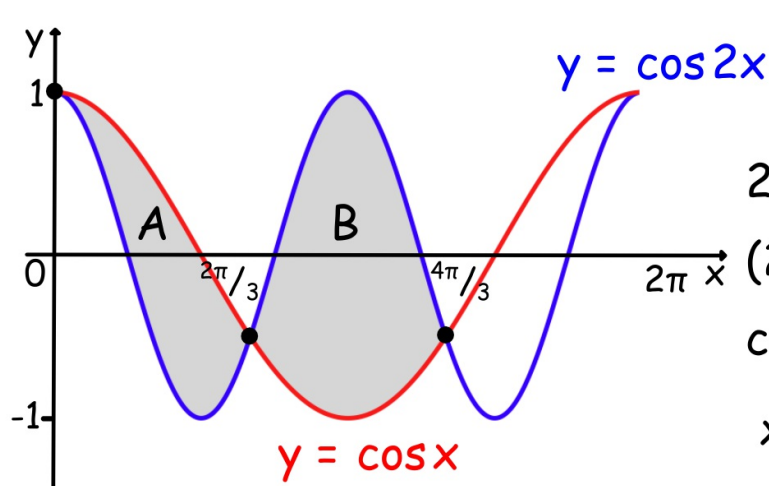
$$= 0 - \left( -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2} \right)$$

$$= 0 + \frac{3\sqrt{3}}{4}$$

$$= \frac{3\sqrt{3}}{4}$$

AREA  $\frac{3\sqrt{3}}{4}$  units<sup>2</sup>

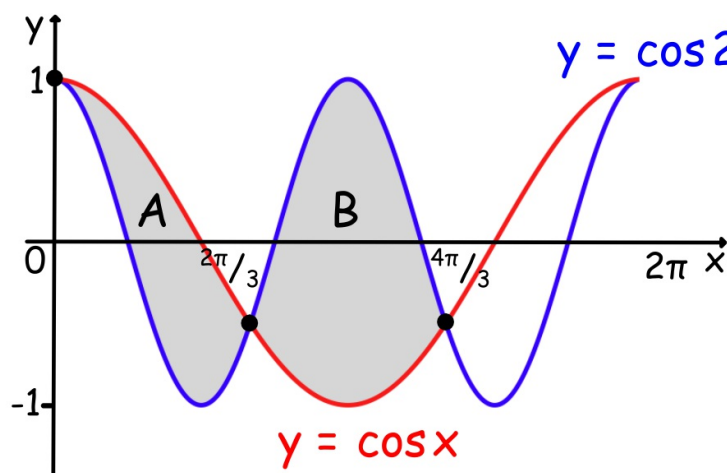




$$\begin{aligned} \cos 2x &= \cos x \\ 2\cos^2 x - 1 &= \cos x \\ 2\cos^2 x - \cos x - 1 &= 0 \\ (2\cos x + 1)(\cos x - 1) &= 0 \\ \cos x &= -\frac{1}{2} \quad \text{or} \quad \cos x = 1 \\ x &= \frac{2\pi}{3}, \frac{4\pi}{3}, \\ &\text{or } x = 0, 2\pi \end{aligned}$$

$$\begin{aligned} &\int_0^{2\pi/3} (\cos x - \cos 2x) dx \\ &= \left[ \sin x - \frac{1}{2} \sin 2x \right]_0^{2\pi/3} \\ &= \left( \sin \frac{2\pi}{3} - \frac{1}{2} \sin \frac{4\pi}{3} \right) - \left( \sin 0 - \frac{1}{2} \sin 0 \right) \\ &= \left( \frac{\sqrt{3}}{2} - \frac{1}{2} \left( -\frac{\sqrt{3}}{2} \right) \right) - \left( 0 - \frac{1}{2} \times 0 \right) \\ &= \left( \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} \right) - 0 \\ &= \frac{3\sqrt{3}}{4} \end{aligned}$$

AREA  $\frac{3\sqrt{3}}{4}$  units<sup>2</sup>



$$\begin{aligned} \cos 2x &= \cos x \\ 2\cos^2 x - 1 &= \cos x \\ 2\cos^2 x - \cos x - 1 &= 0 \\ (2\cos x + 1)(\cos x - 1) &= 0 \\ \cos x &= -\frac{1}{2} \quad \text{or} \quad \cos x = 1 \\ x &= \frac{2\pi}{3}, \frac{4\pi}{3}, \\ &\text{or } x = 0, 2\pi \end{aligned}$$

$$\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (\cos 2x - \cos x) dx$$

$$= \left[ \frac{1}{2} \sin 2x - \sin x \right]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}$$

$$= \left( \frac{1}{2} \sin \frac{8\pi}{3} - \sin \frac{4\pi}{3} \right) - \left( \frac{1}{2} \sin \frac{4\pi}{3} - \sin \frac{2\pi}{3} \right)$$

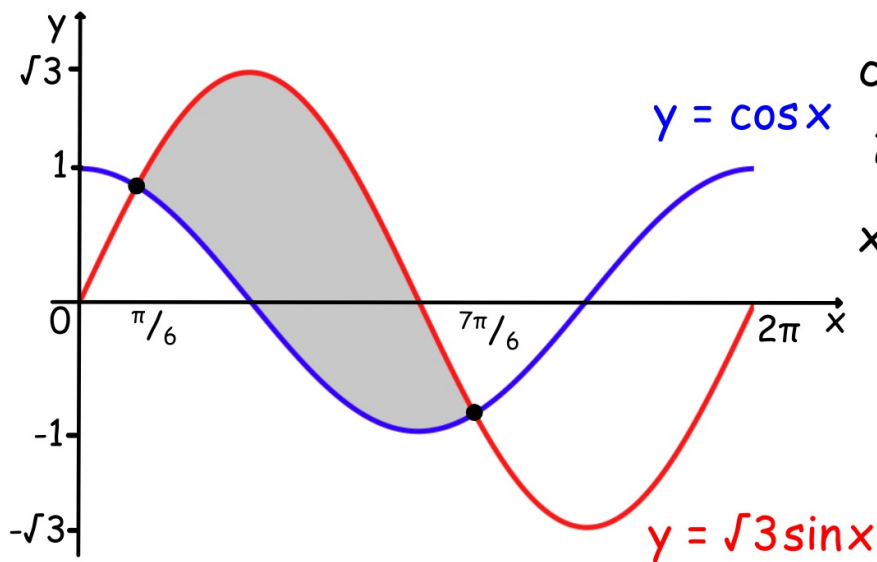
$$= \left( \frac{1}{2} \times \frac{\sqrt{3}}{2} - \left( -\frac{\sqrt{3}}{2} \right) \right) - \left( \frac{1}{2} \left( -\frac{\sqrt{3}}{2} \right) - \frac{\sqrt{3}}{2} \right)$$

$$= \left( \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} \right) - \left( -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2} \right)$$

$$= \frac{3\sqrt{3}}{4} + \frac{3\sqrt{3}}{4}$$

$$= \frac{3\sqrt{3}}{2}$$

AREA  $\frac{3\sqrt{3}}{2}$  units<sup>2</sup>



$$\begin{aligned} \cos x &= \sqrt{3} \sin x \\ \cos x - \sqrt{3} \sin x &= 0 \\ 2 \cos(x + \pi/3) &= 0 \\ x + \pi/3 &= \pi/2, 3\pi/2 \\ x &= \pi/6, 7\pi/6 \end{aligned}$$

$$\begin{aligned} 1 \cos x - \sqrt{3} \sin x &= R \cos(x + a) & R \sin a &= \sqrt{3} \\ &= R \cos x \cos a - R \sin x \sin a & R \cos a &= 1 \\ & & R &= 2 \quad a = \pi/3 \end{aligned}$$

$$\begin{aligned} &\int_{\pi/6}^{7\pi/6} (\sqrt{3} \sin x - \cos x) dx \\ &= \left[ -\sqrt{3} \cos x - \sin x \right]_{\pi/6}^{7\pi/6} \\ &= (-\sqrt{3} \cos 7\pi/6 - \sin 7\pi/6) - (-\sqrt{3} \cos \pi/6 - \sin \pi/6) \\ &= (-\sqrt{3}(-\sqrt{3}/2) - (-1/2)) - (-\sqrt{3} \times \sqrt{3}/2 - 1/2) \\ &= (3/2 + 1/2) - (-3/2 - 1/2) \\ &= 2 + 2 \\ &= 4 \end{aligned}$$

AREA 4 units<sup>2</sup>