Show

(1) $(\cos A + \sin A)(\cos A - \sin A) = \cos 2A$

(2) $\cos^4 A - \sin^4 A = \cos 2A$

(3) $(\sin A + \cos A)^2 = 1 + \sin 2A$

(4) $\cos^2 A (1 + \tan^2 A) = 1$

(5) $\dfrac{1 - \cos^2 A}{1 - \sin^2 A} = \tan^2 A$

(1) $(\cos A + \sin A)(\cos A - \sin A) = \cos 2A$

$(\cos A + \sin A)(\cos A - \sin A)$

$= \cos^2 A + \cancel{\sin A \cos A} - \cancel{\sin A \cos A} - \sin^2 A$

$= \cos^2 A - \sin^2 A$

$= \cos 2A$

(2) $\cos^4 A - \sin^4 A = \cos 2A$

$\cos^4 A - \sin^4 A$

$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$

$= \cos 2A \times 1$

$= \cos 2A$

(3) $(\sin A + \cos A)^2 = 1 + \sin 2A$

$(\sin A + \cos A)(\sin A + \cos A)$

$= \sin^2 A + \sin A \cos A + \sin A \cos A + \cos^2 A$

$= \cos^2 A + \sin^2 A + 2\sin A \cos A$

$= 1 + \sin 2A$

(4) $\cos^2 A (1 + \tan^2 A) = 1$

$\cos^2 A (1 + \tan^2 A)$

$= \cos^2 A + \cos^2 A \tan^2 A$

$= \cos^2 A + \cos^2 A \dfrac{\sin^2 A}{\cos^2 A}$

$= \cos^2 A + \sin^2 A$

$= 1$

(5) $\dfrac{1 - \cos^2 A}{1 - \sin^2 A} = \tan^2 A$

$\dfrac{1 - \cos^2 A}{1 - \sin^2 A}$

$= \dfrac{\sin^2 A}{\cos^2 A}$

$= \tan^2 A$