

Show

$$(1) (\cos A + \sin A)(\cos A - \sin A) = \cos 2A$$

$$(2) \cos^4 A - \sin^4 A = \cos 2A$$

$$(3) (\sin A + \cos A)^2 = 1 + \sin 2A$$

$$(4) \cos^2 A (1 + \tan^2 A) = 1$$

$$(5) \frac{1 - \cos^2 A}{1 - \sin^2 A} = \tan^2 A$$

$$(1) (\cos A + \sin A)(\cos A - \sin A) = \cos 2A$$

$$\begin{aligned} & (\cos A + \sin A)(\cos A - \sin A) \\ = & \cos^2 A + \cancel{\sin A \cos A} - \cancel{\sin A \cos A} - \sin^2 A \\ = & \cos^2 A - \sin^2 A \\ = & \cos 2A \end{aligned}$$

$$(2) \cos^4 A - \sin^4 A = \cos 2A$$

$$\begin{aligned} & \cos^4 A - \sin^4 A \\ = & (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A) \\ = & \cos 2A \times 1 \\ = & \cos 2A \end{aligned}$$

$$(3) (\sin A + \cos A)^2 = 1 + \sin 2A$$

$$\begin{aligned} & (\sin A + \cos A)(\sin A + \cos A) \\ &= \sin^2 A + \sin A \cos A + \sin A \cos A + \cos^2 A \\ &= \cos^2 A + \sin^2 A + 2\sin A \cos A \\ &= 1 + \sin 2A \end{aligned}$$

$$(4) \cos^2 A (1 + \tan^2 A) = 1$$

$$\begin{aligned} & \cos^2 A (1 + \tan^2 A) \\ &= \cos^2 A + \cos^2 A \tan^2 A \\ &= \cos^2 A + \cos^2 A \frac{\sin^2 A}{\cos^2 A} \\ &= \cos^2 A + \sin^2 A \\ &= 1 \end{aligned}$$

$$(5) \frac{1 - \cos^2 A}{1 - \sin^2 A} = \tan^2 A$$

$$\frac{1 - \cos^2 A}{1 - \sin^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$