1. The number of bacteria present in a beaker, during an experiment can be measured using the formula $N(t) = 30e^{1.25t}$ where t is the number of hours passed.

(a) How many bacteria are in the beaker at the start of the experiment?

(b) Calculate the number of bacteria present after 5 hours.

(c) How long will it take for the number of bacteria present to treble?

2. The mass, M grams, of a radioactive isotope after a time of t years, is given by the formula $M = M_o e^{-kt}$ where M_o is the initial mass of the isotope.

In 5 years a mass of 10 grams of the isotope is reduced to 8 grams.

(a) Calculate k.

(b) Calculate the half-life of the substance (time taken for half the substance to decay)

3. A cup of coffee cools according to the law $P_t = P_o e^{-kt}$, where P_o is the initial temperature of the coffee and P_t is the temperature after t minutes.

(a) A cup of coffee cools from 80 °C to 60 °C in a time of 15 minutes. Calculate k.

(b) By how many degrees will the cup of coffee cool in the next 15 minutes?

4. A fire spreads according to the law $A = A_o e^{kt}$ where A_o is the area covered by the fire when it is first measured and A is the area covered after t hours.

(a) If it takes $1^1/_2$ hours for the fire to double in area, find k.

(b) A bush fire covers an area of 800 km^2. If not tackled, calculate the area the fire will cover 4 hours later.

5. The value, V (£million), of a container ship is given by the formula $V = 120e^{-0.065t}$ where t is the number of years after the ship is launched.

(a) Calculate the value of the ship when it was launched.

(b) Calculate the percentage reduction in value of the ship after 6 years.

6. A cell culture grows at a rate given by the formula $y(t) = Ae^{kt}$ where A is the initial number of cells and $y(t)$ is the number of cells after t hours.

(a) It takes 24 hours for 500 cells to increase in number to 800. Find k.

(b) Calculate the time taken for the number of cells to double.

1. $N(t) = 30e^{1.25t}$

(a) $N(0) = 30 \times e^{0} = 30 \times 1 = 30$

(b) $N(5) = 30 \times e^{1.25 \times 5} = 15540.384... = 15540$

(c)
$$30\,e^{1.25t} = 90$$
$$e^{1.25t} = 3$$
$$\log_e e^{1.25t} = \log_e 3$$
$$1.25t \log_e e = \log_e 3$$
$$1.25t \times 1 = \log_e 3$$
$$t = \frac{\log_e 3}{1.25} = 0.8788... \text{ hours}$$
$$= 0.8788... \times 60 \text{ min}$$
$$= \underline{\underline{52.7 \text{ min}}}$$

2. $M = M_o e^{-kt}$

(a)
$$10\,e^{-k \times 5} = 8$$
$$e^{-5k} = 0.8$$
$$\log_e e^{-5k} = \log_e 0.8$$
$$-5k \log_e e = \log_e 0.8$$
$$-5k \times 1 = \log_e 0.8$$
$$k = \frac{\log_e 0.8}{-5}$$
$$= 0.04462...$$
$$= \underline{\underline{0.045}}$$

(b)
$$100\,e^{-0.045t} = 50$$
$$e^{-0.045t} = 0.5$$
$$\log_e e^{-0.045t} = \log_e 0.5$$
$$-0.045t \log_e e = \log_e 0.5$$
$$-0.045t \times 1 = \log_e 0.5$$
$$t = \frac{\log_e 0.5}{-0.04462...}$$
$$= 15.531...$$
$$= \underline{\underline{15.5 \text{ years}}}$$

3. $P_t = P_o e^{-kt}$

(a) $80\, e^{-k \times 15} = 60$

$$e^{-15k} = 0.75$$

$$\log_e e^{-15k} = \log_e 0.75$$

$$-15k \log_e e = \log_e 0.75$$

$$-15k \times 1 = \log_e 0.75$$

$$k = \frac{\log_e 0.75}{-15}$$

$$= 0.01917...$$

$$= \underline{\underline{0.019}}$$

(b) $P_t = P_o e^{-0.019t}$

$$P_{15} = 60 \times e^{-0.01917... \times 15}$$

$$= 45$$

$$60 - 45 = \underline{\underline{15^\circ}}$$

4. $A = A_o e^{kt}$

(a) $100\, e^{k \times 1.5} = 200$

$$e^{1.5k} = 2$$

$$\log_e e^{1.5k} = \log_e 2$$

$$1.5k \log_e e = \log_e 2$$

$$1.5k \times 1 = \log_e 2$$

$$k = \frac{\log_e 2}{1.5}$$

$$= 0.4620...$$

$$= \underline{\underline{0.46}}$$

(b) $A = A_o e^{0.46t}$

$$= 800 \times e^{0.4620... \times 4}$$

$$= 5079.683...$$

$$= \underline{\underline{5080 \text{ m}^2}}$$

5. $N(t) = 120e^{-0.065t}$

(a) $N(0) = 120 \times e^{0} = 120 \times 1 = 120$

(b) $N(6) = 120 \times e^{-0.065 \times 6} = 81.246...$

reduction $120 - 81.246... = 38.753...$

$$\frac{38.753...}{120} \times 100\,\% = 32.294...\,\% = \underline{\underline{32.3\%}}$$

6. $y(t) = A\,e^{kt}$

(a)

$$500\,e^{k \times 24} = 800$$

$$e^{24k} = 1.6$$

$$\log_e e^{24k} = \log_e 1.6$$

$$24k \log_e e = \log_e 1.6$$

$$24k \times 1 = \log_e 1.6$$

$$k = \frac{\log_e 1.6}{24}$$

$$= 0.01958...$$

$$= \underline{\underline{0.020}}$$

(b)

$$500\,e^{0.02t} = 1000$$

$$e^{0.02t} = 2$$

$$\log_e e^{0.02t} = \log_e 2$$

$$0.02t \log_e e = \log_e 2$$

$$0.02t \times 1 = \log_e 2$$

$$t = \frac{\log_e 2}{0.01958...}$$

$$= 35.394...$$

$$= \underline{\underline{35.4 \text{ hours}}}$$

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$P_{15} = 60 \times e^{-0.01917... \times 15}$

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(b)

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14. $y(t) = Ae^{kt}$

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